The Temporal Change of the Geoid Determined from GRACE monthly sets of data

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Abstract: Simple regression analysis is used to extract the temporal change of the geoid from about 12.5 years of GRACE satellite mission data. The analysis is carried out in a global grid of 64.000 points**,** resulting in a map where the geoid changes with time between -4.8 mm/yr and +1.2 mm/yr. The major subductions of the geoid in W. Antarctica and Greenland are caused by ice sheet melting and discharges. There is also a smaller but significant negative change in the Middle East, centred over the Caspian Sea, which could be related to ground water withdrawal. Large positive changes of 1.2 mm/yr over Hudson Bay, NE Canada, and of 1.1 mm/yr in E. Antarctica and of 0.5 mm/yr in Fennoscandia are all due to postglacial rebound. The standard error of the geoid changes is estimated to 0.05 mm/yr.

1 Introduction

The temporal changes of the geoid are caused by mass variations with time in the Earth system, including the atmosphere. An excellent way of studying this phenomenon on a global or regional scale is by the analysis of approximately 12.5 years of monthly computed sets of Earth Gravitational Models (EGM) in the form of spherical harmonics distributed by some geodetic a computing centres. Each of these EGMs is computed from the gravity related data collected by the twin satellite system GRACE. Using these data the change of the geoid with time can be determined by a simple regression analysis as described in Sect. 2. The numerical results are presented in Sect. 3 followed by a discussion in Sect. 4. Section 5 concludes the paper.

2 The regression analysis - theory

Consider the observation equation for the regression

$$a + bt_i = l_i - \varepsilon_i \; ; \quad i = 1, \dots, n, \tag{1}$$

where a and b are the unknown regression parameters, t_i is the epoch of time for recording the observation l_i with error ε_i, and n is the number of observations. If

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one replaces t_i by $\Delta t_i = t_i - \overline{t}$, where $\overline{t}$ is the mean value of all epochs t_i, the set of observation equations can be written as a matrix equation with obvious notations:

$$\begin{bmatrix} 1 & \Delta t_1 \\ .. & .. \\ 1 & \Delta t_n \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} l_1 \\ .. \\ l_n \end{bmatrix} - \boldsymbol{\varepsilon}, \tag{2}$$

with the normal equation

$$\begin{pmatrix} n & 0 \\ 0 & \sum_{i=1}^{n} (\Delta t_i)^2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} l_i \\ \sum_{i=1}^{n} l_i \Delta t_i \end{pmatrix}. \tag{3}$$

The least squares solution becomes

$$\hat{a} = \sum_{i=1}^{n} l_i / n \quad \text{and} \quad \hat{b} = \sum_{i=1}^{n} \Delta t_i l_i / \sum_{i=1}^{n} (\Delta t_i)^2 \tag{4}$$

with standard errors

$$s_{\hat{a}} = s / \sqrt{n} \quad \text{and} \quad s_{\hat{b}} = s / \sqrt{\sum_{i=1}^{n} (\Delta t_i)^2}, \tag{5}$$

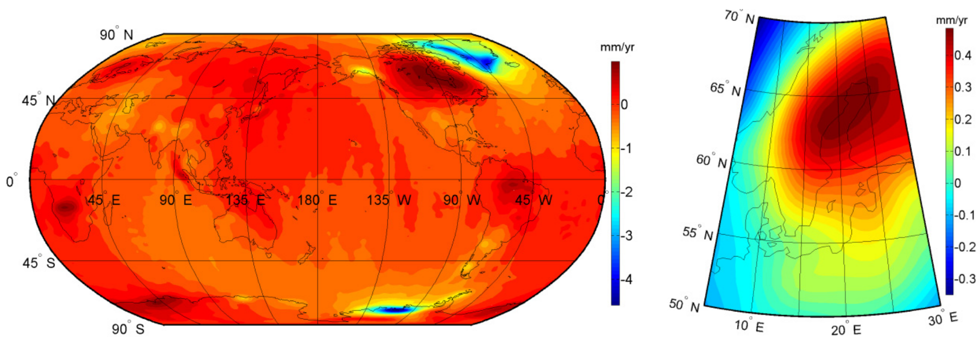
where the variance of unit weight (s) is given by

$$s^2 = \sum_{i=1}^{n} \left(l_i - \hat{a} - \hat{b} \Delta t_i \right)^2 / (n-2). \tag{6}$$

Here $\hat{b}$ yields the temporal change of the set of observations, which is of primary interest for this study.

3 Numerical results

About 12.5 years (between April 2002 and October 2014) of monthly sets of GRACE based EGMs determined at the University of Texas (Bettadpur 2012), complete to degree and order 96, were used for estimating 150 time tagged geoid height observations at each of 64 000 discrete points distributed in a global 1x1-degree grid. Each EGM was filtered for coloured noise by Kusche's (2007) technique with an anisotropic filter. For each point Eq. (4) was applied to estimate the secular, temporal change of the geoid, and the result is plotted in Figs. 1a (global map) and Fig 1b (regional map over Fennoscandia).



(a) ***(b)***

Figure 1. *The secular rate of change of the geoid ($\hat{b}$, using 2.6f):* ***a****) globally,* ***b****) regionally in Fennoscandia for time period of data between April 2002 and October 2014 obtained from linear regression for each point, computed on a 1×1 arc-deg grid of surface points.*

As the data are subject also to possibly harmful long-periodic variations that may provide significantly biased estimates of the secular change, the most prominent periodic signals, stemming from long-periodic tidal frequencies, were also included in some special adjustments, revealing that they have no significant impact on the estimated secular changes, the latter with standard errors of the order of 0.05 mm/yr.

4 Discussions

There are various causes for the secular geoid changes shown in Figs. 1a and 1b that range between -4.8 and +1.2 mm/yr. The largest positive changes in NE Canada, centred over Hudson Bay with a peak of 1.2 mm/a, in Fennoscandia, centred over the Bothnian Bay with a peak of 0.5 mm/yr, and in East Antarctica with 1.1 mm/yr, are all caused by postglacial rebound (or Glacial Isostatic Adjustment) of masses in the lithosphere. The results for Fennoscandia agrees fairly well with previous estimate of Sjöberg (1983) of 0.68 mm/yr derived from land uplift rate data. There are also regions with relatively large geoid uplifts in SW Africa as well as in the Amazonas in S. America of unknown origins. The largest negative changes of the geoid, caused by ongoing melting and discharging of continental ice sheets, can be seen in Greenland and W. Antarctica with peaks of -4.2 and -4.8 mm/yr, respectively. There are also notable subductions of the geoid in SE Alaska, possibly related with the melting of perma frost, and in the Middle East, in particular at the region of the Caspian Sea, likely due to ground water sinking, as well as in southern S. America, possibly related with glacier retreats. See also Sjöberg and Bagherbandi (2017, Sect. 8.7).

5 Conclusions

The study shows that simple regression analysis is a suitable tool to estimate the secular change of the geoid from long records of repeated Earth Gravitational Models, both globally and regionally. The geoid changes range between -4.8 mm/yr and +1.2 mm/yr. The largest negative changes are interpreted as caused by on-going ice mass loss in W. Antarctica and Greenland, while the largest positive changes are due to Glacial Isostatic Adjustments in Laurentia, E. Antarctica and Fennoscandia. The shape and the magnitude of the geoid rate in the last region agree well with previous estimates based on land uplift rates determined from repeated precise levelling and tide gauge data.

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