Is really the "Local Datum" an Obsolete and Old-Fashioned Definition? The catalytic role of the velocity field for the modern geodetic reference frames: The Minimum Kinetic Energy Criterion

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Μηδείς ἀγεωμέτρητος εἰσίτω μου τὴν στέγην
(επιγραφή εις την είσοδον της Ακαδημίας Πλάτωνος)

Let no one ignorant of geometry come under my roof
(an inscription above the door of Plato's Academy)

Abstract: In the present paper we provide an alternative concept for the realization of an optimal reference frame which fits to the local geophysical features of a country or a region. The main idea is based on the implementation of the Minimum Kinetic Energy Criterion (MKEC). The MKEC can be materialized using exploiting 3D, 2D or even in 1D velocities, respectively. The alternative methodology is tested in a GNSS network of 151 stations located in Greece. The results point out that the MKEC criterion reduces more than 50% the initial velocity field and offers a serious advantage compared to the global or a regional Terrestrial Reference Frames (TRFs), respectively. We provide all the necessary mathematical equations for its successful application.

Keywords: reference frames, kinetic energy, velocities, tectonic plates, optimality

1. Introduction

The local geodetic reference system or local datum dominated the geodetic science from the 19th century (e.g. Helmert 1880) till the end of the 20th century. The local datum was the basis for all the geodetic studies in a specified area. In most of the cases, the area was a country, or more recently (after the 2nd World War) a continent e.g. ED 50 in Europe. The local datum was closely related with the triangulations (e.g. Fotiou and Livieratos 2000), especially with the high order one (1st and 2nd order, respectively -Rossikopoulos 1999). In fact, the local datum comprised a set of conventions, principles and definitions regarding some fundamental geodetic quantities (Fotiou and Livieratos 2000).

The main objective of the local datum was to reduce as possible the influence of the gravity field to the observations. The measurements were mainly angles, distances, and azimuths, respectively, which refer to the physical Earth's surface. However the computations should be done in the map projection plane where one can apply easier the associated formulas (e.g. Torge and Mueller 2012). Thus, one should properly reduce the observations from the physical surface to the projection plane. The reduction procedure demands good knowledge of the deflections of the vertical (ξ,η) and the geoid (N) (Heiskanen and Moritz 1967). The local datum's construction philosophy leads to the minimization of the aforementioned gravity related quantities, fitted in an optimal sense to the country or continent.

The departure of the space techniques, like the Doppler (TRANSIT system, e.g. Torge and Mueller 2012) and especially GNSS changed deeply the route of the geodetic practice. The geodesists (or nowadays even a simple user) having a single GNSS device can determine (with a certain accuracy) the 3D position of any arbitrary point in earth's surface. Hence, there is a great need of a reference system which provides in a global sense accurate 3D position and in addition, precise information regarding the dynamic change of the position (taking into account the velocities). The International Terrestrial Reference Frame (ITRF, Altamimi et al. 2002) tend to give mm and sub-mm/yr accuracy for the position and the velocity, respectively. This kind of “globalization” seems to cancel out the initial need of the local datum definition:

Why one should reduce accurate and consistent observations?
Who really needs cumbersome computations? and finally:
Who needs more “theory” than “practice”?

Along with the ITRF, there are several regional TRFs realizations for each continent. In Europe is maintained the European Terrestrial System of 1989 (ETRS89, Gubler and et al. 1992), which is the officially accepted geodetic TRF for the European continent. Practically, ETRS89 is based on ITRF in a way that “follows the motion of the Eurasian plate”. In order to achieve the latter requirement, the ETRS89 velocities are computed (initially with respect to an ITRF version) by subtracting the velocities derived from a tectonic plate model for the Eurasian region.

The ETRS89 application delivers relatively small velocities for the central and northern Europe. On the other hand, for countries like Greece, Turkey or Italy do not give any significant advantage regarding the reduced velocities (Figure 1).

The velocity field (the dynamical behavior of a particular region) of a TRF seems to play the same role as the gravity quantities in the old datums. The geodesists, the surveyors and the cartographers would like to handle coordinates which “do not severely change through time”. The reasons are quite obvious: The users, the state

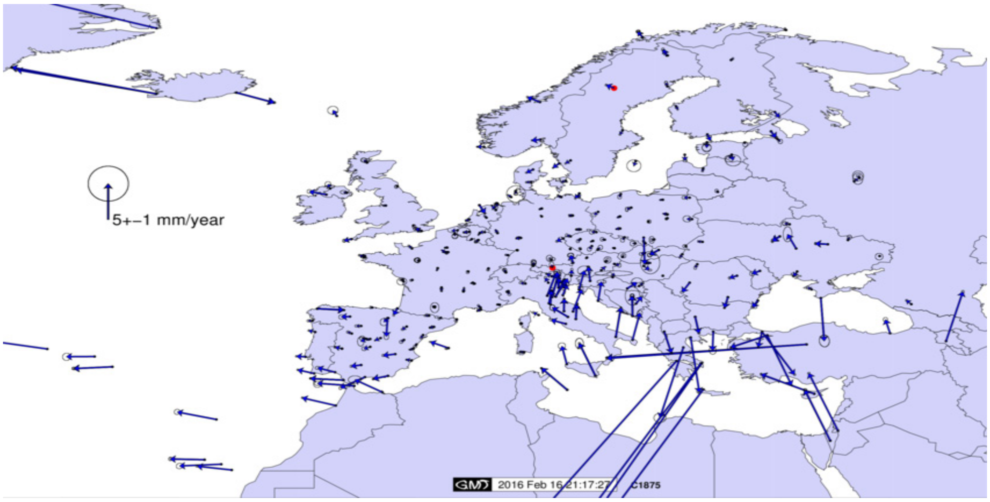

Figure 1: *The 2D velocity field of the European Permanent Network with respect to the Eurasian plate (www.epncb.oma.be)*

agencies and the professionals would prefer a mapping reference system which is rather stable as possible.

A global or a regional reference system (frame) answers relatively well the need of precise positioning. However it/they is/are blind to local effects and probably does/do not offer any significant advantage in many cases.

2. How can we derive an optimal geodetic system (frame)?

One can immediately ask: How can we define a geodetic reference system which fits optimally to the dynamic behavior of a country/region? Let us "travel to the past", recalling the optimal fitting procedure followed for the local datums. The nucleus of the mathematical procedures, was the estimation of transformation parameters (mainly translations) which minimized in an optimal way the geoidal undulations or/and the deflections of the vertical. The transformation parameters were applied to a global reference frame in order to create the new local datum. The optimization criterion can be described as follows, pointwise (e.g. Heikanen and Moritz 1967):

$$\sum_{i=}^{n} N_i^2 = \min \tag{1a}$$

$$\sum_{i=}^{n} \xi_i^2 = \min \tag{1b}$$

$$\sum_{i=}^{n} \eta_i^2 = \min \tag{1c}$$

where N is the geoidal undulation, and ξ, η the deflections of the vertical, respectively. Mathematically speaking, the optimal local datum was derived by minimizing the associated gravity quantities in least squares sense.

2.1 The modern case of optimization

Our approach is based on the velocity transformation between two reference systems. This can be expressed through the following formula, pointwise (Ampatzidis 2011, Ampatzidis et al. 2011):

$$\mathbf{v}_i^{ORF} = \mathbf{v}_i^{TRF} + \mathbf{E}_i\dot{\boldsymbol{\theta}} \tag{2}$$

where $\mathbf{v}_i^{ORF} = \begin{bmatrix} v_x^{ORF} & v_y^{ORF} & v_z^{ORF} \end{bmatrix}_i^T$ the 3D velocities with respect to the Optimal Reference Frame (ORF), $\mathbf{v}_i^{TRF} = \begin{bmatrix} v_x^{TRF} & v_y^{TRF} & v_z^{TRF} \end{bmatrix}_i^T$ the velocities with respect to a global or regional TRF, $\dot{\boldsymbol{\theta}} = \begin{bmatrix} \dot{t}_x & \dot{t}_y & \dot{t}_z & d\dot{s} & \dot{r}_x & \dot{r}_y & \dot{r}_z \end{bmatrix}^T$ the seven Helmert parameter rates (three translation, one differential scale and three orientation rates, respectively). Finally, $\mathbf{E}_i$ is the associated design matrix :

$$\mathbf{E}_i = \begin{bmatrix} 1 & 0 & 0 & x_i^{TRF} & 0 & -z_i^o & y_i^{TRF} \\ 0 & 1 & 0 & y_i^{TRF} & z_i^{TRF} & 0 & -x_i^{TRF} \\ 0 & 0 & 1 & z_i^{TRF} & -y_i^{TRF} & x_i^{TRF} & 0 \end{bmatrix}$$

where $x_i^{TRF}, y_i^{TRF}, z_i^{TRF}$ the triplet of the coordinates of an arbitrary point i with respect to a TRF. Applying the optimization criterion (e.g. Dermanis 1987):

$$\left(\mathbf{v}^{ORF}\right)^T \mathbf{P}\left(\mathbf{v}^{ORF}\right) = \min \tag{3}$$

we finally get:

$$\hat{\dot{\boldsymbol{\theta}}} = -\left(\mathbf{E}^T\mathbf{P}\mathbf{E}\right)^{-1}\mathbf{E}^T\mathbf{P}\mathbf{v}^{TRF} \tag{4}$$

and

$$\mathbf{v}^{ORF} = \mathbf{v}^{TRF} + \mathbf{E}\hat{\dot{\boldsymbol{\theta}}} \tag{5}$$

where $\mathbf{v}^{ORF} = \begin{bmatrix} \mathbf{v}_i^{ORF} & \cdots & \mathbf{v}_n^{ORF} \end{bmatrix}^T$, $\mathbf{v}^{TRF} = \begin{bmatrix} \mathbf{v}_i^{TRF} & \cdots & \mathbf{v}_n^{TRF} \end{bmatrix}^T$, $\mathbf{E} = \begin{bmatrix} \mathbf{E}_i^T & \cdots & \mathbf{E}_n^T \end{bmatrix}^T$ $\mathbf{P}$ is a proper weight matrix (e.g. the inverse of the covariance matrix of the velocity errors). Equation (4) refers to the estimated parameters for the transition of a global or regional TRF to the optimal LRF. Equation (5) is used for the estimation of the new velocities of the ORF. From another point of view, the optimization criterion leads to ORF whose total kinetic energy is minimized. It is the so-called Minimum Kinetic Energy Criterion (MKEC, Ampatzidis 2011).

2.2 Different options for the MKEC approach

One can use different options for the MKEC in relation to which velocities aims to minimize. We have the following options:

1. The case of the 3D velocities: This holds for the case which we want to minimize the total 3D velocities (both horizontal and vertical). Since the translations and the orientations are strongly correlated for small areas (e.g. for a single country), one should use only orientations or only translations. The use of three translations can be considered as the estimation of a single set of Euler Pole Parameters (EPPs) for the area (Ampatzidis et al. 2011).
2. The 2D (horizontal) case: In the 2D case we take into account only the horizontal part of the velocities, with respect to the latitude and longitude (horizontal geodetic velocities). In order to proceed, we should modify properly the design matrix **E** because the optimization criterion should be expressed in the ellipsoid. This practically leads to a Molodensky-type transformation for the curvilinear velocities; for the mathematical formulations please see Okeke (1998) (the models should modified for the velocity case, accordingly) and Ampatzidis (2011). The 2D case is suitable for the areas with intense tectonic activities (earthquakes, volcanoes, faults) like Greece, Japan and Italy.
3. The 1D case: This is an interesting approach in the cases of ares which the land uplift is significant. For examples in the countries/areas which the Glacial Isostatic Adjustment (GIA). For instance, in the Northern Europe, Baltic countries, Canada, Antarctica, Southern America.

3. Results

The MKEC was implemented for the Hellenic area, following the second option (using exclusively horizontal velocities (Bitharis et al. 2016b). The GNSS network consists of 151 stations. The MKEC criterion is implemented to the ITRF2008 and ETRF2000 velocity fields, respectively. Table 1 gives the statistical behavior of the horizontal velocities according to each velocity field. Figures 2 and 3 depict the initial and the MKEC velocity fields respectively.

Table 1 and Figures 2 and 3 represent the results after the application of the MKEC. We can imply that the MKEC defines a velocity field for the Greek area, reducing the mean horizontal velocity from 23.9 (ETRF2000) to 10.8 mm/yr (MKEC-ETRF2000), respectively, corresponding to a reduction of almost 55%. The kinetic energy of the MKEC reference frame is almost 5 times lower than the ETRF2000 case. In addition, the maximum horizontal velocity is 23.8 mm/yr (instead of 38.9 mm/yr for ETRF2000). Practically, the velocity field is significantly smoothed.

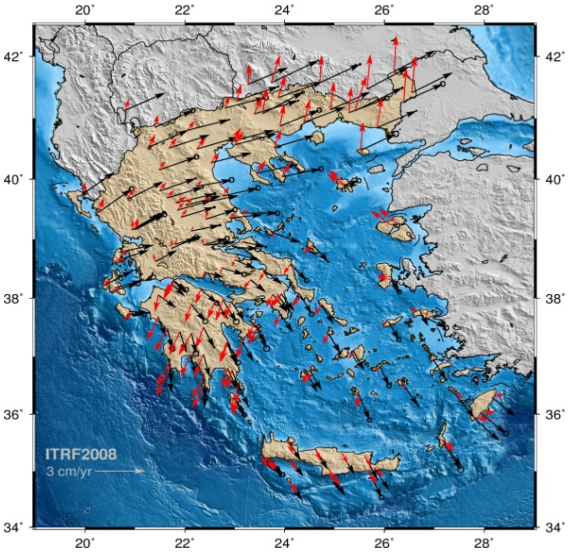

Figure 2*: The ITRF2008 (black arrows) and the MKEC-wise velocities (red arrows) for the MKEC-wise velocities (red arrows) for the network of 151 stations (Bitharis et al. 2016b).*

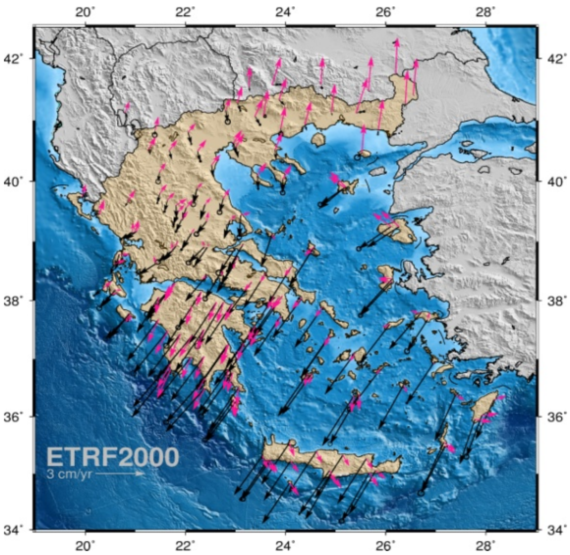

Figure 3*: The ETRF2000(black arrows) and the MKEC-wise velocities (pinkarrows) for the MKEC-wise velocities (red arrows) for the network of 151 stations (Bitharis e t al. 2016b).*

Table 1*: The statistics of the velocities for the Hellenic area. Units are mm/yr, except of the kinetic energy which is expressed in mm^2/yr^2 (Bitharis et al. 2016).*

	ITRF2008	ITRF2008(MKEC)	ETRF2000	ETRF2000(MKEC)
min	4.8	0.5	0.9	0.6
max	32.2	23.6	38.9	23.8
σ	5.7	4.7	12.0	4.7
mean	18.0	9.7	20.7	9.7
rms	18.9	10.8	23.9	10.8
median	15.5	8.8	26.0	8.8
kinetic energy	53811.2	17681.7	86414.5	17493.3

However, one can argue that the derived velocity field is not homogeneous. This is true, since the MKEC approach does not imply any pure geophysical meaning (like the case of ETRS89). One can also suggest the use of multiple Euler Poles for the Greek area. Indeed, many researchers (e.g. McKenzie 1972, McClusky et al. 2000, Reilinger et al. 2006) conclude that the area is divided into smaller microplates. This is a convenient choice for the geophysical studies. Nevertheless, this option can cause problems for the geodetic applications. The borders of the microplates are approximately defined (e.g. with an accuracy of few kilometers). Thus near the borders of the microplates the application of the Euler Poles it is possible to give distorted results. Furthermore, the estimation of the Euler Pole Parameters (e.g. Drewes 1982) in limited areas is problematic due to the strong correlations.

Another interesting finding is the fact that the ETRS89 does not show any advantage comparing to ITRF2008 velocity field. On the contrary, the kinetic energy of the ITRF2008 is 38% lower than the ETRF2000 case.

4. Conclusions and further work

In the present paper we provide a brief discussion regarding the modern aspect of the local geodetic datum definition. Revisiting the mathematical formulation from the classical geodetic literature, we provide an alternative strategy for the optimal local geodetic reference system realization, using the least squares principal. We applied the MKEC criterion to the Greek area (which presents intense and inhomogeneous geodynamic behavior). We find that the 2D MKEC offers significant advantages, mainly reducing severely the mean horizontal velocity field of the Greek area and its total kinetic energy, respectively.

The MKEC could be applied to other areas with intense tectonic behavior (e.g. Italy, Turkey, New Zealand etc.). In addition the 1D case (using height velocities only) could be applied in regions of intense vertical motions (like Canada, Scandinavia).

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