Italian quasi-geoid estimate at Politecnico di Milano: A historical overview

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1. Introduction

The estimate of a high precision quasi-geoid is a relevant task in modern Geodesy. Quasi-geoid can be used to compute the geoid, the equipotential surface of the Earth gravity field which is close to the mean ocean surface.

As it is well known, the geoid can be used in combination with radar-altimetry data to get ocean currents. Furthermore, GPS observations combined with geoid estimates can give orthometric heights. This is of particular relevance, since in this way orthometric heights can be computed in a faster and cheaper way than using spirit levelling, although with lower precision (which is however sufficient in many practical applications).

Thus, the geodetic community has done many efforts in order to compute reliable estimate of the quasi-geoid based on new and refined global geopotential models and precise global DTMs that are nowadays available for terrain effect computation and reduction.

In this paper, a historical review of quasi-geoid computation in Italy performed at Politecnico di Milano is given. The quasi-geoid estimated at Politecnico di Milano have been derived by using collocation and the "remove-restore" technique. The first reliable estimate was computed in 1995 and was named ITALGEO95 (Barzaghi et al., 1996).

Since then, improved solutions have been derived by improving the gravity database, the DTM and using the most updated global geopotential models to account for the long-wavelength components of gravity and geoid.

Among other quasi-geoid/geoid estimates performed in Italy, the quasi-geoids computed at Politecnico di Milano have been adopted by the Istituto Geografico Militare (IGM) as the official reference surface to be used in Italy to derive orthometric heights from GPS ellipsoidal heights.

2. The gravity and the GPS/leveling databases in Italy

The Italian gravity database used in estimating the quasi-geoid was mainly supplied by the Servizio Geologico Nazionale. These data have been validated and improved including data sets from other national agencies and data centers.

The Italian gravity data set, used for the ITALGEO95 estimate (Barzaghi et al., 1996) contained 105695 gravity values.

This data set has been successively enlarged introducing 4232 new gravity data in the area corresponding to Slovenia (Figure 1). In this way, the gravity data gap in this area, which is very close to the Italian boundaries, was filled thus avoiding possible mismodelling in the quasi-geoid estimate in the Friuli Venezia Giulia area. This data set was used in computing ITALGEO99 (Barzaghi et al., 2002).

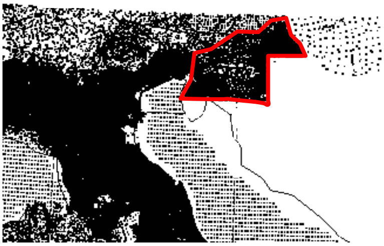

Figure 1: *The gravity data set in the Slovenia area (red line block)*

A further improvement in the gravity data base was realized for the ITALGEO05 estimate (Barzaghi et al, 2007). New data were included and an improved outlier rejection based on collocation was adopted.

To reduce boundary effects in the Alpine region, the area covered by gravity has been enlarged by one degree more in each direction, considering data supplied by DMA (USA Defence Mapping Agency), BGI (Bureau Gravimetric International) and the gravity data collected in the context of the Brennero geoid project (Barzaghi et al., 2003).

This improved gravity database, consisting of 310660 gravity values, covers the area $5° \le \varphi \le 20°$, $35° \le \lambda \le 48°$ with a mean density of 20″.

As one can see in Figure 2, the gravity data coverage is quite dense in most part of Italy, even though in the central part there are areas presenting a poor coverage. However, these are areas of limited extension that should not affect remarkably the quasi-geoid estimate. An important large area having a poor data coverage is the Alpine region. This mostly affects the reliability of the estimated geoid in this area since high frequency gravity signals are not properly sampled (e.g. the gravity signal implied by the Ivrea body). This situation will be hopefully improved in the near future since there are plans for densifying the gravity data over the Alps through aerogravimetry.

GPS/levelling data were also collected in the framework of geoid computation. As it is well known, by subtracting the orthometric height from the ellipsoidal height, one can get point-wise values of the geoid. These data are used for checking the gravity geoid precision or for getting an integrated geoid estimate based both on gravity and GPS/levelling data.

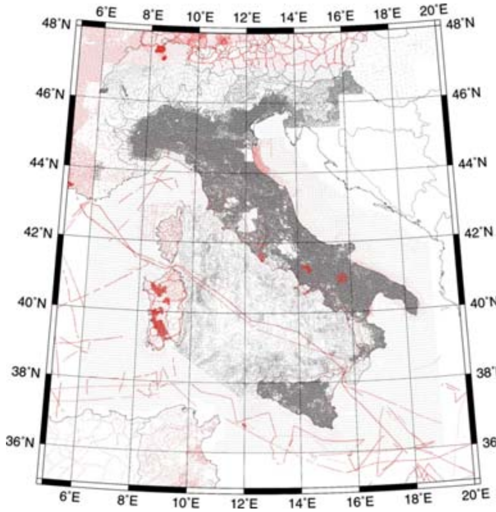

Figure 2: *The gravity database used in estimating the ITALGEO05 solution (red dots = new gravity values; grey dots = gravity data used to get ITALGEO99).*

Such database has been provided by IGM (Surace, 1997) that constantly improved it. In 1995, the available GPS/levelling undulations were 187. Nowadays, this number is increased to 1068. Furthermore, most of the levelling lines have been recently re-measure and more lines have been added to those constituting the 1995 database. Thus, this data set (see Figure 7) can be considered a valuable and reliable tool in geoid control and computation.

3. The DTM database used in geoid computations

A reliable and detailed DTM is strictly necessary in geoid estimation to account for terrain effect.

In computing the ITALGEO95 solution, a 250 m × 250 m resolution DTM was compiled by merging different DTMs over the estimation area. The Italian 7.5″ × 10″ DTM by Carrozzo et al. (1982) was integrated outside the Italian boundaries with the Austrian, French, German, Swiss and ETOPO5U (1998) DTMs and on sea with the Morelli bathymetry (Morelli et al., 1975). The same DTM was also used in the ITALGEO99 solution after a reliability check based on the 100 m × 100 m Italian DTM supplied by IGM. All these DTMs were based on map digitalization and were merged by weighted average.

In the ITALGEO05 solution the new SRTM3 (Shuttle Radar Topography Mission) DEM was adopted. It has a spatial resolution of 3″ × 3″ (about 100m × 100m) and it covers homogeneously the whole Earth land territory. It is derived from SAR measurements, so it represents not only the terrain heights, but also man-made structures. The nominal precision is 16 m in the horizontal component and 20 m in the vertical component. The SRTM3 gives no information over the sea and inside narrow valleys, where there are hidden areas.

The SRTM3 represents an important data set, because it has a high spatial resolution and it is in a unique reference system. For these reasons it was chosen as a basis for a new DTM data set. This DTM has been integrated with other digital models to fill the gaps in the valleys and it has been completed with bathymetric data as detailed in the following:

- the Italian DTM used for ITALGEO99 has been used to fill the gaps in the Italian land region and for the of bathymetry near the coasts, where good resolution digitalised bathymetry is available;
- a new $1' \times 1'$ NOAA bathymetry has been used in deep seas (https://128.160.23.42/dbdbv/dbvquery.html);
- the GTOPO30 DTM has been considered for the remaining areas with no data (http://edc.usgs.gov/products/elevation/gtopo30/gtopo30.html).

The area on which the new DTM was assembled is $33° < \varphi < 50°$ and $3° < \lambda < 22°$, 2° on each side larger than the area containing the gravity database. The resulting grid has a regular geographical mesh of $3'' \times 3''$ (Borghi et al., 2007).

The precision of the new Italian DTM has been assessed considering height information contained in the gravity database. Usually, to each gravity station an observed orthometric height is associated. In Table 1, these values are compared with those estimated using the SRTM derived DTM and the DTM on which the ITALGEO99 geoid is based (we considered a subset of the Italian gravity data base containing 90203 points).

Table 1: *Statistics of the difference between gravity database heights and DTMs estimates*

	ITALGEO99 DTM	ITALGEO05 DTM
n	90203	90203
E (m)	0.90	0.47
σ (m)	43.83	25.32
Min (m)	–1500.00	–821.98
Max (m)	1177.70	907.94

The statistics of the differences show that the SAR based DTM gives sharply better results than the one used in ITALGEO99 computation. This is due both to the higher resolution and to the uniform quality of the SRTM data. So, the more detailed and reliable SRTM DTM can be considered a remarkable improvement of the previous Italian DTM.

4. Global geopotential models in the Italian area

Different global geopotential models have been used and tested in estimating the Italian quasi-geoid. In the ITALGEO95 solution, the OSU91A (Rapp, 1994) has been used to model the low-frequency component in gravity and geoid. At that time, it was commonly adopted and considered the best available model.

The statistics of the raw gravity data and those of the gravity residuals, after model reduction, are shown in Table 2.

Table 2: *Statistics of raw and OSU91A reduced gravity data (ITALGEO95 database)*

	Δg_0	$\Delta g_0 - \Delta g_{OSU91A}$
n	105695	105695
E (mGal)	11.87	–7.57
σ (mGal)	62.86	33.47
Min (mGal)	348.82	309.67
Max (mGal)	–578.43	–197.34

The remarkable reduction in the mean and in the st.dev. of the raw data proves the model quality. However, OSU91A has evident mismodellings in the Mediterranean area, e.g. the gravity signal of the Corsica Island is completely missed.

In the subsequent estimates, improved different model have been tested to define the best fit model over the Central Mediterranean. In computing the ITALGEO99, the new EGM96 (Lemoine et al., 1998) and GPM98CR models (Wenzel 1998) were compared and both used to get two different solutions. As one can see (Table 3), the GPM98CR gave the best results. This is quite an expected result since EGM96 is complete to degree and order 360 while GPM98CR reaches degree and order 720.

Table 3: *Statistics of raw, EGM96 and GPM98CR reduced gravity data (ITALGEO99 database)*

	Δg_0	$\Delta g_0 - \Delta g_{EGM96}$	$\Delta g_0 - \Delta g_{GPM98CR}$
n	109927	109927	109927
E (mGal)	11.92	–6.71	–6.07
σ (mGal)	61.71	30.75	25.20
Min (mGal)	–162.36	–253.33	–200.55
Max (mGal)	269.71	187.95	164.85

GPM98CR was also adopted in last estimation procedures, even though new EIGEN models have been made available. As a matter of facts, after the ITALGEO99 issue, the space missions CHAMP and GRACE gave new contributions to the knowledge of the Earth gravity field. So, the new available models were tested to evaluate possible improvements in modelling the low frequency components of gravity filed. The EIGEN-CG03C model (Förste et al, 2005), based on 860 days of CHAMP solutions and 376 days of GRACE solutions combined with 0.5° × 0.5° surface data, was selected and tested in the Italian geoid computation area. The results of the comparison showed that the GPM98CR model still was, at that time, the best fit model, giving the best statistics of the residuals (see Table 4).

Table 4: *Statistics of raw and EIGEN-CG03C reduced gravity data (ITALGEO99 database)*

	Δg_0	$\Delta g_0 - \Delta g_{EIGEN\text{-}CG03C}$
n	109927	109927
E (mGal)	11.92	–7.15
σ (mGal)	61.71	32.01
Min (mGal)	–162.36	–254.50
Max (mGal)	269.71	225.10

The situation is nowadays remarkably changed. As it is well known, the EGM2008 model has been recently released (Pavlis et al., 2008). It is an ultra high degree model, as it is up to degree 2160, and it represents a sharp improvements with respect to all the previous mentioned models. This can be clearly seen in the statistics of the residuals of gravity (listed in Table 5).

Table 5: *Statistics of raw and reduced gravity data using EGM2008 and GPM98CR geopotential models (ITALGEO05 database)*

	Δg_0	$\Delta g_0 - \Delta g_{EGM2008}$	$\Delta g_0 - \Delta g_{GMP98CR}$
n	310660	310660	310660
E (mGal)	11.52	–5.22	–6.58
σ (mGal)	63.93	18.38	23.99
Min (mGal)	–162.55	–243.34	–228.65
Max (mGal)	269.71	119.49	168.01

This new model will be possibly used in refinements of the geoid estimate in Italy. However, in the opinion of the author, the computations based on EGM2008 cannot be performed straightforwardly, e.g. by applying collocation in the context of the "remove-restore" procedure. As it will be seen in the next paragraph, the covariance function of the residuals cannot be properly modeled using the standard covariance models (Tscherning and Rapp, 1974, Tscherning et al., 1994). Thus, modifications to the techniques commonly adopted in geoid computation are in order to take full advantage of the new high resolution EGM2008 model.

5. From ITALGEO95 to ITALGEO05 and the integrated Italian quasi-geoid

The quasi-geoids computed since 1995 were four: ITALGEO95, ITALGEO99, ITALGEO05 and ITALGEO95I. The first three estimates are based on gravity data only while the last one is an estimate based on a combination of gravity and GPS/leveling data. They have been computed using the "remove-restore" procedure and collocation. As mentioned before, the "remove-restore" procedure was performed using either OSU91A (as for ITALGEO95) or GPM98CR and the DTMs described in paragraph 3. In all the estimates, the Residual Terrain Component was evaluated by means of the TC program of the GRAVSOFT package (Tscherning et al., 1994). The reference DTM used in RTC evaluation, was estimated by moving average as applied to the detailed DTM. The proper window size of the moving average varied from one solution to the other as a function of the adopted global geopotential model. It was selected by testing different window sizes, choosing the one giving the best statistics in the residual gravity (i.e. observed gravity minus the model component and the RTC effect).

5.1. The gravimetric quasi-geoid estimates

The residual quasi-geoid components of the three gravimetric based estimates were obtained via Fast-Collocation (Bottoni and Barzaghi, 1993). Thus, residual gravity data have been gridded on a 3'× 3' regular geographical grid. Quasi-geoid estimates were given on the same grid. In this context, the empirical and the model covariance functions, which tune and define the solutions, are the relevant issues. Figure 3 represents the empirical and the model covariances leading to the ITALGEO95 estimate. Model covariance has been interpolated using the COVFIT program of the GRAVSOFT package (Tscherning et al., 1994) (the same program was also used in the other cases).

The agreement between the empirical values and the model covariance is quite good and the same holds for the ITALGEO99 solution based on the EGM96 global geopotential model (see Figure 4).

On the contrary, while using the GPM98CR model to reduce the data for estimating the ITALGEO99 solution, an irregular structure of the empirical covariance is obtained (see Figure 5). These irregularities cannot be properly fitted by the stan-

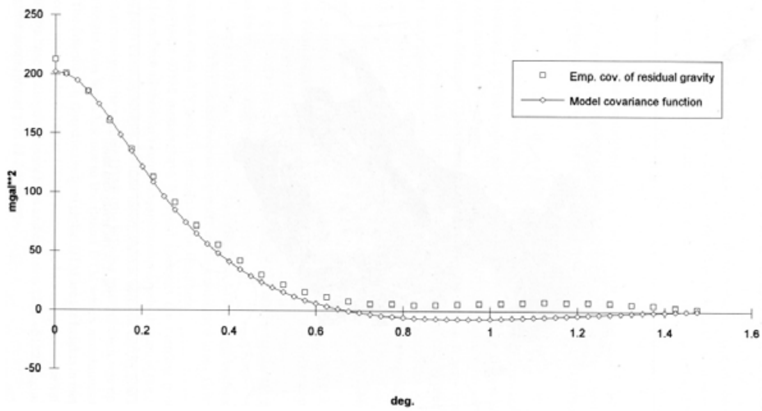

Figure 3: *The empirical and the model covariance of gravity residuals of the ITALGEO95 solution based on OSU91A*

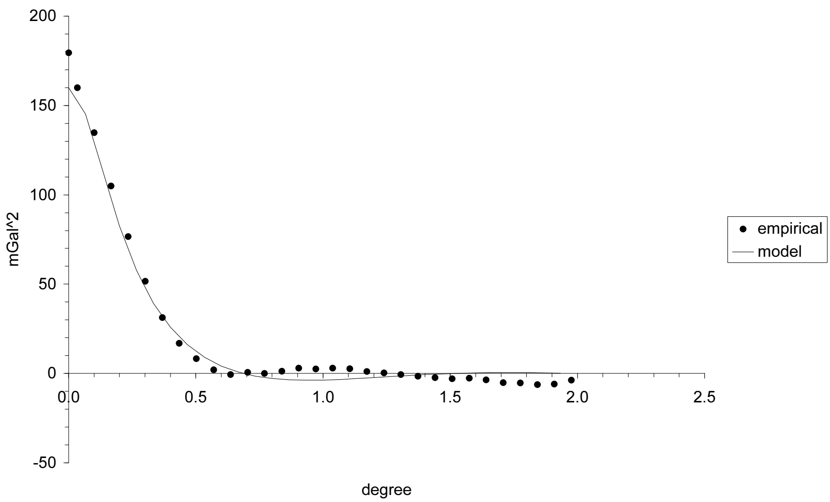

Figure 4: *The empirical and the model covariance of gravity residuals of the ITALGEO99 solution based on EGM96*

dard covariance models which always display a regular and smooth pattern. Nevertheless, the model is still able to reproduce, at least, the mean structure of the empirical values. However, it must be stressed that the agreement between model and empirical covariance is quite poor. The same behavior can be seen in the covariance structure of the ITALGEO05 solution which is still based on the GPM98CR model.

This structure of the empirical estimates is indeed what one can expect to have. While increasing the model degree (properly tuning the RTC effect computation),

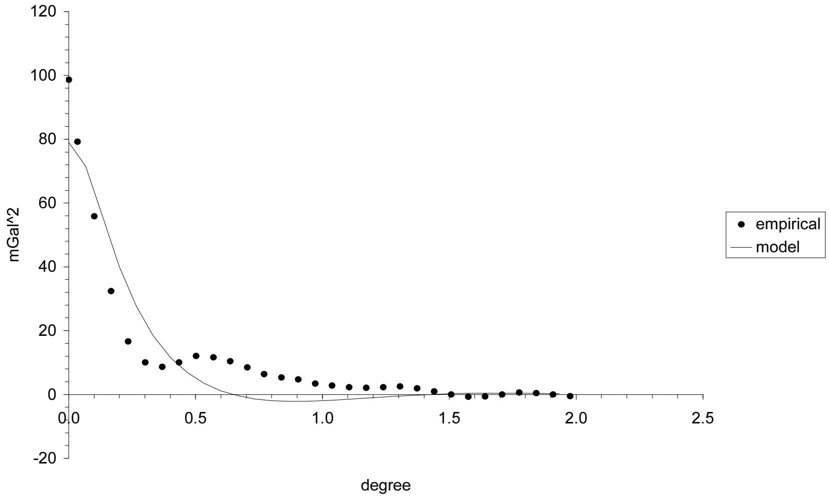

Figure 5: *The empirical and the model covariance of gravity residuals of the ITALGEO99 solution based on GPM98CR*

residuals contain higher frequencies which reflect into shorter correlation length of the empirical covariances. Also, irregularities in the empirical covariance can derive from the structure of the residual gravity which tends to become a white noise signal.

This behavior is confirmed when reducing the data with EGM2008. In this case, the situation becomes even worse than the one displayed in Figure 5 (see Figure 6). A sharp decrease in the correlation length is clearly visible and the empirical covariance is practically a white noise covariance. As a consequence, these values can be hardly fitted using the standard model covariance functions. Thus, as already mentioned, collocation cannot be applied straightforwardly and this demands for new approaches in estimating local residual geoids in forthcoming computations.

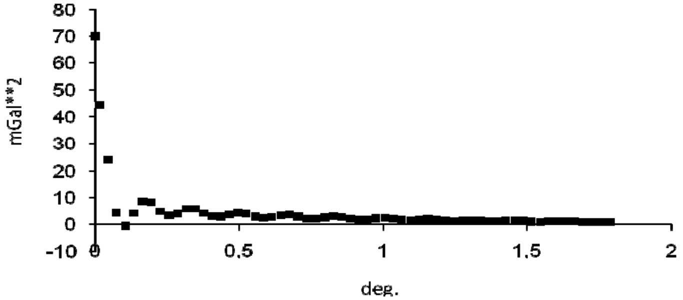

Figure 6: *The empirical covariance of gravity residuals of the Italian gravity data based on EGM2008*

The precision of the three estimated quasi-geoids have been tested by comparison with GPS/leveling data. In this paper, results are shown for the two last estimates, i.e. ITALGEO99, based on GPM98CR, and ITALGEO05 (the comparison between ITALGEO95 and GPS/leveling can be found in Barzaghi et al., 1996). After datum shift estimation (formula (2-176b) in Heiskanen and Moritz, 1993) and outliers rejection using a significance threshold of 1%, the following results were obtained (see Tables 6 and 7).

Table 6: *Statistics of the residuals between ITALGEO99 and $N_{GPS/lev}$ after datum shift*

	ITALGEO99		
	Peninsular Italy	Sicily Island	Sardinia Island
n	932	42	48
E (m)	0.00	0.00	0.00
σ (m)	0.12	0.05	0.06
Min (m)	–0.30	–0.08	–0.15
Max (m)	0.31	0.11	0.14

Table 7: *Statistics of the residuals between ITALGEO05 and $N_{GPS/lev}$ after datum shift*

	ITALGEO05		
	Peninsular Italy	Sicily Island	Sardinia Island
n	957	43	47
E (m)	0.00	0.00	0.00
σ (m)	0.11	0.06	0.04
Min (m)	–0.29	–0.12	–0.09
Max (m)	0.29	0.10	0.11

The statistics obtained from the two quasi-geoids are almost the same. However there is a lower number of rejected points when using ITALGEO05 (21 versus 46 when using ITALGEO99). It is expected that some of the points considered as outliers with ITALGEO99 are actually consequence of a mismodelling due to a lack of gravity values or to a poor DTM resolution. Hence, this comparison proves that the ITALGEO05 estimate is significantly better than ITALGEO99. This is also confirmed by comparing Figure 7 and Figure 8 where residuals with respect to ITALGEO99 and ITALGEO05 respectively are classed. In some areas, the agreement between the GPS/leveling geoid estimate and the gravimetric quasi-geoid has been clearly improved (e.g. the coastline of Liguria).

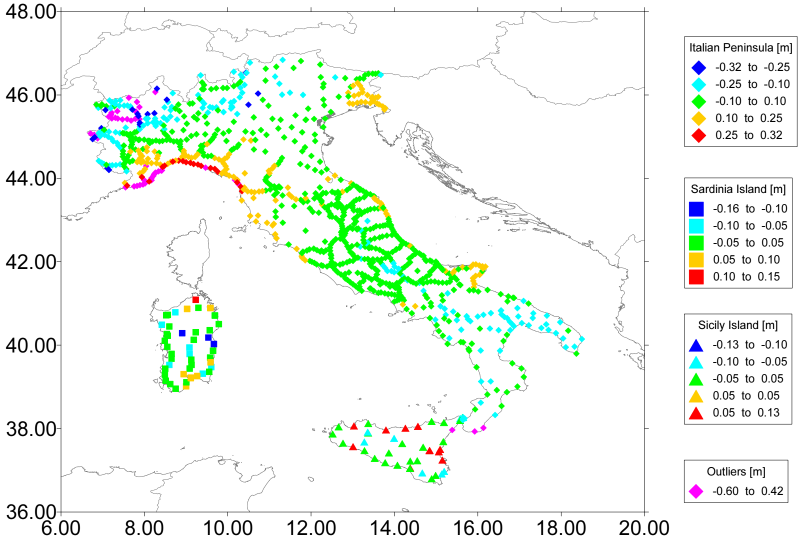

Figure 7: *Residuals between ITALGEO99 and $N_{GPS/lev}$ after datum shift*

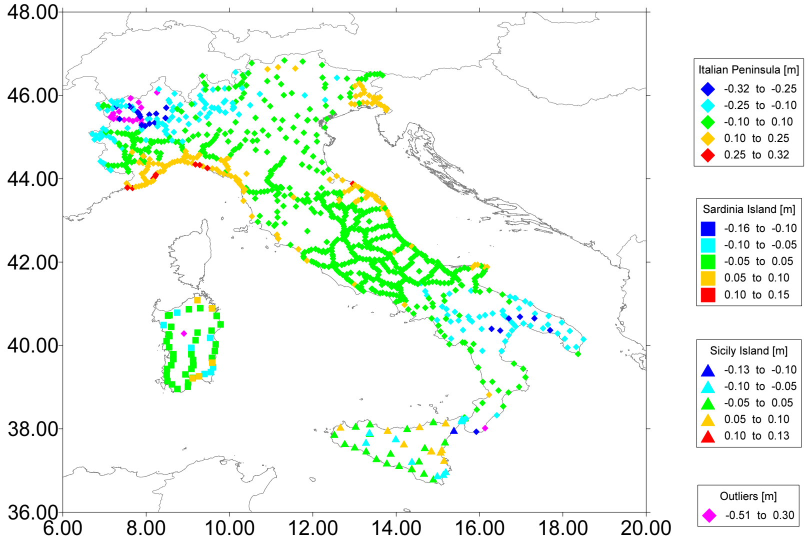

Figure 8: *Residuals between ITALGE05 and $N_{GPS/lev}$ after datum shift*

5.2. The integrated Italian quasi-geoid

In geodetic applications, collocation allows using observations on different functionals of the anomalous potential T(P) to get an estimate of any functional of T(P) itself (Moritz, 1980). By means of this general scheme, one can, for instance, estimate the geoid undulation N(P) from observed Δg, (ξ, η) and N(P) values.

As a matter of fact, any geodetic observation can be written in a linearized form as

$$\delta H_j = (A\delta\chi)_j + L^{(P_j)}(T) + n_j\,, \quad j = 1,\ \ldots,\ m \tag{1}$$

where A is a known matrix, $\delta\chi$ is a vector of unknown parameters to be estimated, $L^{(P_j)}(T)$ is a linear functional of T and n_j is a white noise signal. By applying the Wiener-Kolmogorov principle, one can get in P the collocation estimate of any linear functional $\hat{L}^{(P)}(T)$ of T

$$\hat{L}^{(P)}(T) = \sum L^{(P)} L^{(P_i)} C_{TT}(P,P_i)\left[C_{\delta H\delta H}\right]_{ij}^{-1}(\delta H - (A\delta\hat{\chi}))_j \tag{2}$$

where

$$\delta\hat{\chi} = (A^t C_{\delta H\delta H}^{-1} A)^{-1} A^t C_{\delta H\delta H}^{-1}\delta H$$

$$(C_{\delta H\delta H})_{ij} = L^{(P_i)} L^{(P_j)} C_{TT}(P_i,P_j) + \sigma_n^2 I$$

and

$$C_{TT}(P,Q) = C_{TT}(\psi_{PQ}) = \frac{1}{8\pi}\int_\sigma d\sigma_{P'}\int_0^{2\pi} d\sigma_{Q'} T(P')T(Q'), \quad (\psi_{PQ} = \psi_{P'Q'})$$

is the covariance function of T(P) which can be estimated from observed values of any functional of T(P) (Moritz, 1980).

This methodological scheme has been used to merge gravity and GPS/levelling data in order to get the integrated geoid estimate in the Italian area.

GPS/levelling data were transformed in the gravimetric geoid reference system (formula (2-176b) in Heiskanen and Moritz, 1993). Gravity and undulation data were then reduced for the GPM98CR global geopotential model effect and for the RTC component. The general scheme discussed above is then applied to

$$\delta H = \begin{bmatrix} \Delta g_r \\ N_r \end{bmatrix} + \begin{bmatrix} n_g \\ n_N \end{bmatrix} = \begin{bmatrix} L_g(T_r) \\ L_N(T_r) \end{bmatrix} + \begin{bmatrix} n_g \\ n_N \end{bmatrix} \tag{3}$$

(in this case, no $\delta\chi$ parameters are present in the observation equations) with

$$\begin{aligned} \Delta g_r(P_k) &= \Delta g_{obs}(P_k) - \Delta g_M(P_k) - \Delta g_{RTC}(P_k) \\ N_r(P_j) &= N_{obs}(P_j) - N_M(P_j) - N_{RTC}(P_j) \end{aligned} \qquad P_k, P_j = \textit{observation points}$$

where $\Delta g_M,\ N_M$ are the global geopotential model components and $\Delta g_{RTC},\ N_{RTC}$ are the residual terrain effect components.

Using (2), one can compute an integrated estimate of the residual undulation, namely $\hat{N}_r$. As usual, the final geoid undulation estimate in a point P is then obtained by restoring the model and the RTC components.

The empirical covariance and the model covariance function of Δg_r used in the computation of $\hat{N}_r$ are shown in Figure 5.

The computation of formula (2) has been efficiently implemented using gridded gravity data. As a matter of fact, the $C_{\delta H\delta H}$ matrix is partitioned as it follows

$$C_{\delta H\delta H} = \begin{bmatrix} C_{\Delta g\Delta g} & C_{\Delta gN} \\ C_{N\Delta g} & C_{NN} \end{bmatrix}$$

where $C_{\Delta g\Delta g}$ and C_{NN} are the (square) auto-covariance matrix of Δg_r and N_r respectively, while $C_{\Delta gN}$ and $C_{N\Delta g}$ are the cross-covariances between these two functionals. If we have gridded gravity data, $C_{\Delta g\Delta g}$ has a Toeplitz/Toeplitz structure (Bottoni and Barzaghi, 1993) and can be inverted using fast and efficient algorithms (Kailath and Wax, 1983). Thus, the estimation formula (2) can be computed considering the partititioned structure of $C_{\delta H\delta H}$, taking advantage of the fast computation of the inverse of $C_{\Delta g\Delta g}$ (which is usually much larger than C_{NN}). The integrated geoid estimate is then obtained according to the following formulas

$$\begin{aligned} \hat{N}_r &= \sum \left[C_{N\Delta g}, C_{NN} \right] \left[C_{\delta H\delta H} \right]^{-1} \begin{bmatrix} \Delta g_r + n_g \\ N_r + n_r \end{bmatrix} = \\ &= \sum \left[C_{N\Delta g}, C_{NN} \right] \left[C_{\delta H\delta H} \right]^{-1} \begin{bmatrix} X_g \\ X_N \end{bmatrix} = \\ &= \sum \left[C_{N\Delta g}, C_{NN} \right] \begin{bmatrix} s_g \\ s_N \end{bmatrix} \end{aligned} \tag{4}$$

$$\begin{bmatrix} C_{\Delta g\Delta g} & C_{\Delta gN} \\ C_{N\Delta g} & C_{NN} \end{bmatrix} \begin{bmatrix} s_g \\ s_N \end{bmatrix} = \begin{bmatrix} X_g \\ X_N \end{bmatrix}$$

$$\begin{cases} s_N = (C_{NN} + C_{ng} C_{gg}^{-1} C_{gN})^{-1} (X_N - C_{Ng} C_{gg}^{-1} X_g) \\ \\ s_g = C_{gg}^{-1} (X_g - C_{gN} s_N) \end{cases} \tag{5}$$

This solution method was implemented as a windowed procedure where only gravity and undulation observations within a 3° distance from the computation point were taken into account as input data (this distance is set according to the covariance structure of the gravity residuals).

The gravity data base was the same gridded gravity used in computing ITALGEO05. The available GPS/levelling data have been split into two distinct sets: 768 Data Points (DP), used in the estimation procedure and 300 Check Points (CP) used to assess the quality of the estimated solution.

The two subsets are plotted in Figure 9.

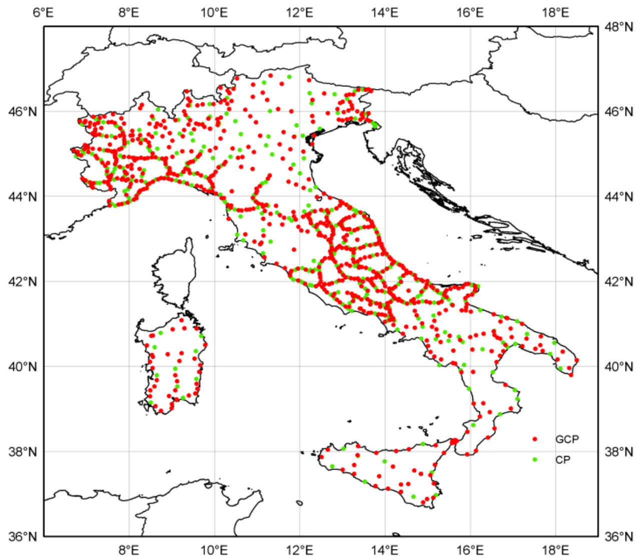

Figure 9*: The GPS/levelling database divided into DP(red) and CP(green) points*

Statistics of the residuals on the CPs and their spatial distribution are described in Table 8 and Figure 10 respectively.

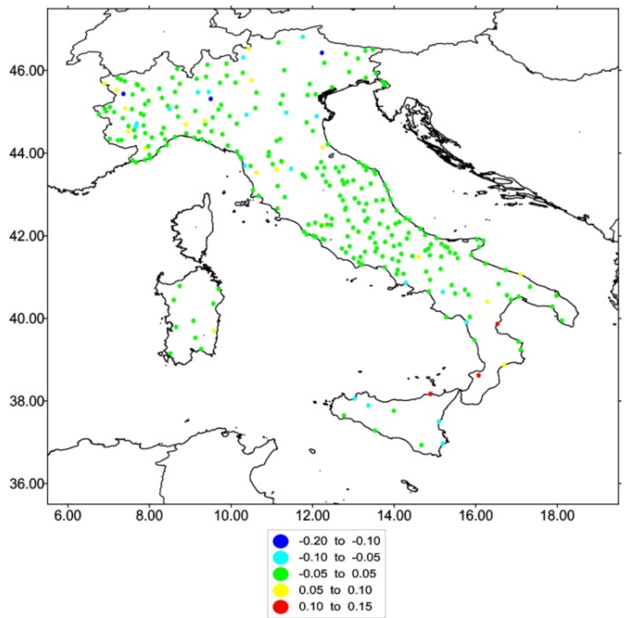

Figure 10: *Residuals between GPS/lev and the integrated geoid estimate on CPs (m)*

Table 8: *Statistics of the residuals between GPS/lev and the integrated estimate on CPs*

	$N_{GPS/lev} - \hat{N}$ [m]
n	300
E (m)	0.00
σ (m)	0.04
Min (m)	–0.18
Max (m)	0.15

The improvement with respect to the gravimetric quasi-geoid is remarkable. In this solution, the standard deviation is uniformly at the 4 cm level. This value is attained by the ITALGEO05 solution in the Sardinia Island only.

Thus, the integrated estimate, even though implemented according to a windowed procedure, is able to give a much more precise solution. Since, in the future, large sets of heterogeneous data will became available, it seems advisable to develop collocation based solutions which proved to be feasible and reliable.

6. Conclusions

In this paper, the long road describing geoid estimation in Italy has been presented. The improvements in the computed solution proved to be strongly related to the increments in data availability and precision.

The gravity data base was increased from one solution to the other thus allowing a more detailed description of the high frequency content of the geoid. However, there are still important areas, such as the Alps, having a quite poor gravity data coverage which demands for further data collection efforts.

The GPS/leveling dataset in Italy became denser and much more reliable due to the valuable efforts made by IGM. Thus, this kind of data, used in the beginning for testing purposes only, was included in the geoid estimate in the framework of a collocation solution. The so called integrated geoid solution is nowadays the best geoid estimate in Italy which is currently adopted by IGM as the official geoid to be used in transforming ellipsoidal heights into orthometric heights.

Finally, it must be mentioned the strong impact that global geopotential models have on the final geoid estimate. As they represent nearly the 90% (or more) of the total undulation, they are of extreme importance in computing a reliable solution. In the last twenty years, they have had an impressive improvement, both in term of accuracy and precision. Particularly, the EGM2008 model gives geoid estimates that are nearly equivalent to some high resolution local geoids (e.g. the ITAL-

GEO05 estimate, Barzaghi and Carrion, 2009). This is a remarkable step forward in global gravity filed modeling which also demands for new theoretical and numerical procedures to be used in computing local very high resolution geoid estimates.

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