The evolution of geodetic methods for the determination of strain parameters for earth crust deformations

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Summary

The methods for the determination of crustal deformation parameters such as principal strains, dilatation and shear, are critically reviewed. Apart from the historical evolution of the method emphasis is given in several of their particular aspects, such as the method of observation adjustment, the interpolation of the estimated displacements, their domain of application (plane, three-dimensional space, sphere, ellipsoid, natural earth surface), the problem of datum definition and the related invariance of strain parameters, the separation of deformation from relative rigid motion of tectonic blocks and finally the quality assessment of the obtained results.

1. Introduction

The determination of changes in the shape of the earth is of paramount importance in geophysics in particular for the study of tectonic activity related to earthquakes. Geodesists have developed a vivid interest in the subject even in the early days when observational accuracy could not ensure reliable estimates of point displacements on the earth crust within a reasonably short time interval of a few years. The advent of the space geodesy era and in particular the widespread availability of GPS equipment gave a new life to the topic and extensive research activity is going on implementing a number of different methodologies. Here we will explore the development of various approaches to geodetic determination of crustal deformation from the early days up to now, with emphasis on their methodological characteristics.

Geodesy provides information of a discrete character such as displacements or velocities of control points, while deformation is a spatially continuous phenomenon. Therefore the main criterion for the classification of the various approaches is the explicit or implicit method of spatial interpolation of the information at hand. A second important aspect is whether deformation is studied for the 3-dimensional earth body, which is in fact the only "real" deformation, or some other 2-dimensional entity. The latter choice is necessitated by the fact that observations are carried on the surface of the earth as well as by the fact that the geophysical

origins of horizontal deformation (due mainly to secular tectonic plate motion) are different from those of vertical deformation (due to periodic effects related to annual variation in the earth water cycle or to secular motion associated with post-glacial uplift). For this reason geodesists prefer to study not the deformation of the readily accessible natural earth surface, but rather the deformation of its projection on a "horizontal" surface, such as a plane for a small region, a cartographic projection plane, the sphere or the reference ellipsoid. Since the isolated study of vertical motion though of great geophysical interest poses no methodological challenges to the geodesist, we will confine our present study to methods for the determination of horizontal deformation. We must mention though an alternative approach the so called "integrated geodesy" approach, where geometric displacements are associated with accompanying changes in gravity. In practice such changes are predominantly due to vertical displacements and thus the topic will be altogether excluded from our exposition.

Starting with the simplest case, planar deformation is described by the deformation mapping $\mathbf{x}' = \mathbf{f}(\mathbf{x})$ which corresponds to the Cartesian coordinates $\mathbf{x}$ of any point P at an initial epoch t, its coordinates $\mathbf{x}'$ at a later epoch t'. Locally the function $\mathbf{f}$ is represented by its local linear approximation, the deformation gradient $\mathbf{F} = \partial\mathbf{f}/\partial\mathbf{x} = \partial\mathbf{x}'/\partial\mathbf{x}$. It is in some cases convenient to replace this with the displacement gradient $\mathbf{J} = \partial\mathbf{u}/\partial\mathbf{x} = \partial(\mathbf{x}' - \mathbf{x})/\partial\mathbf{x} = \mathbf{F} - \mathbf{I}$, where $\mathbf{u} = \mathbf{x}' - \mathbf{x}$ are the point displacement vector. In place of $\mathbf{F}$ or $\mathbf{J}$ geophysicists prefer the strain tensor $\mathbf{E}$ associated with variation of the length element from ds at t to ds' at t', through $ds'^2 - ds^2 = 2d\mathbf{x}^T\mathbf{E}d\mathbf{x}$, and since $d\mathbf{x}' = \mathbf{F}d\mathbf{x}$, $ds^2 = d\mathbf{x}^T d\mathbf{x}$, $ds'^2 = d\mathbf{x}'^T d\mathbf{x}'$, it follows that $\mathbf{E} = \frac{1}{2}(\mathbf{F}^T\mathbf{F} - \mathbf{I}) = \frac{1}{2}(\mathbf{J} + \mathbf{J}^T + \mathbf{J}^T\mathbf{J})$. Neglecting second order terms in the small quantities $\mathbf{J}$, the infinitesimal strain tensor is used instead: $\mathbf{E}_{\text{inf}} = \frac{1}{2}(\mathbf{J} + \mathbf{J}^T) \approx \mathbf{E}$. In place of the elements of $\mathbf{E}_{\text{inf}}$ a number of strain parameters which are invariant under particular changes of reference system (see e.g., **Jaeger**, 1969, **Malvern**, 1969, **Fung**, 1977, **Eringen**, 1980, **Livieratos**, 1978, **Livieratos**, 1979, **Livieratos**, 1980, **Dermanis** & **Livieratos**, 1983, **Caspary**, 1987). These are the dilatation Δ, the shear components γ_1, γ_2 and the rotation ω, related to the elements of $\mathbf{E}_{\text{inf}}$ through

$$\mathbf{E}_{\text{inf}} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{12} & \varepsilon_{22} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \Delta + \gamma_1 & \gamma_2 \\ \gamma_2 & \Delta - \gamma_1 \end{bmatrix}, \tag{1}$$

so that $\Delta = \varepsilon_{11} + \varepsilon_{22}$, $\gamma_1 = \varepsilon_{11} - \varepsilon_{22}$ and $\gamma_2 = 2\varepsilon_{12}$. The displacement gradient matrix can be analyzed into a symmetric and antisymmetric part so that

$$\mathbf{J} = \frac{1}{2}(\mathbf{J} + \mathbf{J}^T) + \frac{1}{2}(\mathbf{J} - \mathbf{J}^T) = \mathbf{E}_{\text{inf}} + \mathbf{\Omega} =$$

$$= \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{12} & \varepsilon_{22} \end{bmatrix} + \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \Delta+\gamma_1 & \gamma_2 \\ \gamma_2 & \Delta-\gamma_1 \end{bmatrix} + \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix}, \qquad (2)$$

where $\Delta = J_{11} + J_{22}$, $\gamma_1 = J_{11} - J_{22}$, $\gamma_2 = J_{12} + J_{21}$ and $\omega = \frac{1}{2}(J_{12} - J_{21})$. Thus the 4 parameters Δ, γ_1, γ_2, ω constitute an equivalent set to the 4 elements of $\mathbf{J}$. Th shear components γ_1, γ_2 are replaced by the invariant maximum shear strain $\gamma = \sqrt{\gamma_1^2 + \gamma_2^2}$ and the associated direction angle $\varphi = \frac{1}{2}\arctan(-\gamma_1/\gamma_2)$. Some times the principal direction of contraction $\beta = \varphi + 45^\circ = \frac{1}{2}\arctan(-\gamma_2/\gamma_1)$ is given in the literature as the "direction of γ".

To get rid of the particular time interval $\Delta t = t' - t$ the strain parameters Δ, γ_1, γ_2, ω are divided by Δt to obtain the corresponding time derivatives $\dot{\Delta}$, $\dot{\gamma}_1$, $\dot{\gamma}_2$, $\dot{\omega}$ (strain rate parameters). Thus we implicitly assume that the displacement gradient if a linear function of time $\mathbf{J} = \mathbf{J}_0 + t\dot{\mathbf{J}}$, so that its derivative $\dot{\mathbf{J}}$ and the strain rate parameters are constant with respect to time. In this case γ is replaced bt the invariant parameter $\dot{\gamma} = \sqrt{\dot{\gamma}_1^2 + \dot{\gamma}_2^2} \neq d\gamma/dt = \gamma^{-1}(\gamma_1\dot{\gamma}_1 + \gamma_2\dot{\gamma}_2)$. Another set of strain parameters stemming from the diagonalization of the strain matrix $\mathbf{E} = \mathbf{R}(-\theta)\mathbf{\Lambda}\mathbf{R}(\theta)$, are the maximum and minimal principal strains, $e_{\max} = \Lambda_{11}$ and $e_{\min} = \Lambda_{22}$, respectively and the direction angle θ of $e_{\max}$, typically replaced by their "infinitesimal" counterparts based on the diagonalization $\mathbf{E}_{\inf} = \mathbf{R}(-\theta_{\inf})\mathbf{\Lambda}_{\inf}\mathbf{R}(\theta_{\inf})$. In fact $\Delta = e_{\max} + e_{\min}$ and $\gamma = e_{\max} - e_{\min}$.

As a rigorous alternative to the infinitesimal strain approach **Biagi** & **Dermanis** (2006, 2009) have introduced the singular values $\lambda_{\max}$, $\lambda_{\min}$ of the deformation gradient matrix $\mathbf{F}$ as the fundamental parameters from which other strain parameters can be rigorously derived. These are the positive roots of the common eigenvalues of $\mathbf{F}^T\mathbf{F} = \mathbf{R}(-\theta)\mathbf{\Lambda}^2\mathbf{R}(\theta)$ and $\mathbf{F}\mathbf{F}^T = \mathbf{R}(-\theta')\mathbf{\Lambda}^2\mathbf{R}(\theta')$ and are the diagonal elements of the diagonal matrix $\mathbf{\Lambda}$ appearing in the singular value decomposition $\mathbf{F} = \mathbf{R}(-\theta')\mathbf{\Lambda}\mathbf{R}(\theta)$. Additional parameters are the direction angles of the principal direction, θ in the original and θ' in the second epoch reference system. The principal strains are $e_{\max} = (\lambda_{\max}^2 - 1)/2$ and $e_{\min} = (\lambda_{\min}^2 - 1)/2$, dilatation becomes $\Delta = \lambda_{\max}\lambda_{\min} - 1$, maximum shear strain $\gamma = (\lambda_{\max} - \lambda_{\min})/\sqrt{\lambda_{\max}\lambda_{\min}}$ and its direction angle is $\varphi = \theta - \delta$ where δ is an auxiliary angle determined from $\sin\delta = \sqrt{\frac{1}{2}\left(1 + \gamma/\sqrt{\gamma^2+4}\right)}$ and $\cos\delta = \sqrt{\frac{1}{2}\left(1 - \gamma/\sqrt{\gamma^2+4}\right)}$. The difference $\Delta\theta = \theta' - \theta$ replaces the rotation angle ω of the approximate infinitesimal approach.

2. The early days and the linear model for the deformation function

The first spatial interpolation method for displacements has been based on the linearity assumption that $\mathbf{x}' = \mathbf{F}\mathbf{x} + \mathbf{c} = \mathbf{x} + \mathbf{J}\mathbf{x} + \mathbf{c}$, with constant gradient $\mathbf{F} = \mathbf{I} + \mathbf{J}$ and displacement $\mathbf{c}$, so that the displacements $\mathbf{u} = \mathbf{x}' - \mathbf{x} = \mathbf{J}\mathbf{x} + \mathbf{c}$ are linear functions of the coordinates $\mathbf{x}$ over a particular region under study, while the strain or strain rate parameters are constant within that region. When a larger region is under study where the above assumption cannot hold, it is fragmented into smaller regions where $\mathbf{J}$ and the subsequent strain parameters are constant, a scheme that in mathematics would be characterized as representation (interpolation) by step functions. The simplest choice of sub-regions is non-overlapping triangles of a geodetic control network, a method that has been later characterized as the "finite element" method with triangular elements. From the 6 displacement components $\mathbf{u}_A$, $\mathbf{u}_B$, $\mathbf{u}_C$ at the triangle vertices A, B, C, it is possible to determine uniquely the 6 elements of $\mathbf{J}$ and $\mathbf{c}$. In order to determine the strain parameters the constant displacement $\mathbf{c}$ is eliminated by forming differences, e.g. $\mathbf{u}_B - \mathbf{u}_A = \mathbf{J}(\mathbf{x}_B - \mathbf{x}_A)$, $\mathbf{u}_C - \mathbf{u}_B = \mathbf{J}(\mathbf{x}_C - \mathbf{x}_B)$ and the displacement gradient is determined from

$$\mathbf{J} = \left[\mathbf{u}_B - \mathbf{u}_A \;\vdots\; \mathbf{u}_C - \mathbf{u}_B\right]\left[\mathbf{x}_B - \mathbf{x}_A \;\vdots\; \mathbf{x}_C - \mathbf{x}_B\right]^{-1}. \tag{3}$$

As reported by **Rikitake** (1976) this method has been put in use in Japan by **Terada & Miyam**e (1929) and **Tsuboi** (1933). Since Rikitake's exposition it has become popular mainly due to its simplicity and is still in use by many researchers, e.g., **Harada & Shimura** (1979), **Fujii** & **Nakane** (1983), **Kakkuri** & **Chen** (1990), **Feigl** et al. (1993), **Jouanne**, **Menard** & **Jault** (1994), **Martinod** et al. (1996), **Li**, **Miyashita** & **Fujii** (1997), **Walpersdorf** et al. (1998), **Ferhat**, **Feigl**, **Ritz** & **Souriau** (1998), **Sue** et al. (2000), **Haas**, **Gueguen**, **Scherneck**, **Nothnagel** & **Campbell** (2000), **Fukuda** et al. (2000), **Puglisi**, **Bonforte** & **Maugeri** (2001), **Sagiya**, **Nishimura**, **Iio** & **Tada** (2002), **Bock** et al. (2003), **Dogăn**, **Ergintav, Demirel**, **Çakmak** & **Özener** (2003), **Grafarend** & **Voosoghi** (2003), **Fujii** (2003), **Puglisi** & **Bonforte** (2004), **Mohamed** (2005), **Nishimura** & **Hashimoto** (2006), **Bayer** et al. (2006), **Masson** et al. (2007), **Cai** & **Grafarend** (2007a), **Marotta** & **Sabadini** (2008).

Dermanis (1994) gave a finite element method deriving strain parameters and their covariances directly from the observed 3 distances in trilateration surveys. **Welsch** (1983) presented a finite element method utilizing angle and side length variations both in triangles and larger sub-networks implementing side conditions for shared sides and angle closures. A 3-dimensional finite element method with quadrilateral elements has been used by **Kiamehr** & **Sjöberg** (2005).

The main disadvantage of the method is the fact that a minimal amount of in-

formation is used for the determination of strain parameters and thus it is impossible to detect within each triangle from the results themselves a violation of the assumption of constant strains. Strain parameter covariances calculated by application of the law of error propagation on the covariances of the input displacements reflect the uncertainties in the input information but ignore completely the interpolation error. If neighbouring triangles have strain estimates which are very similar, this is an indication that the interpolating hypothesis holds true to a satisfactory extend. On the other hand large strains in isolated triangles, with small strains in neighbouring triangles, may not reflect local deformation but rather relative motion misinterpreted as deformation, e.g. in the case where the triangle vertices lie on different sides of a fault.

A second line of development is associated with the early analysis of triangulation results in particular with studies at the seismic zone around the Saint Andreas fault in California. The first influential work in the English literature is that of **Frank** (1966). To understand Frank's and subsequent methods we need to express observed temporal variations of azimuths, angles and distances in a geodetic control network as functions of strain parameters. If s_{AB} is the distance between two points A and B and $\mathbf{n}_{AB}$ the unit vector from A to B it holds that $\mathbf{x}_{AB} \equiv \mathbf{x}_B - \mathbf{x}_A = s_{AB}\mathbf{n}_{AB}$ at epoch t and (assuming $\mathbf{J}$ to be constant within the region of interest) $\mathbf{x}'_{AB} = \mathbf{x}_{AB} + \mathbf{J}\mathbf{x}_{AB} = s_{AB}(\mathbf{n}_{AB} + \mathbf{J}\mathbf{n}_{AB})$ at the second epoch t'. The distance $s_{AB} = (\mathbf{x}_{AB}^T \mathbf{x}_{AB})^{1/2}$ becomes $s'_{AB} = s_{AB} + \delta s_{AB} = (\mathbf{x}'^T_{AB}\mathbf{x}'_{AB})^{1/2} = = [(\mathbf{x}_{AB} + \mathbf{J}\mathbf{x}_{AB})^T (\mathbf{x}_{AB} + \mathbf{J}\mathbf{x}_{AB})]^{1/2}$, and the azimuth $a_{AB} = \arctan[(\mathbf{x}_{AB})_1 / (\mathbf{x}_{AB})_2]$ becomes $a'_{AB} \equiv a_{AB} + \delta a_{AB} = \arctan[(\mathbf{x}_{AB} + \mathbf{J}\mathbf{x}_{AB})_1 / (\mathbf{x}_{AB} + \mathbf{J}\mathbf{x}_{AB})_2]$. Ignoring second order terms in the small elements of $\mathbf{J}$ the distance and azimuth variations become

$$\frac{\delta s_{AB}}{s_{AB}} \approx J_{11}\sin^2 a_{AB} + J_{22}\cos^2 a_{AB} + (J_{12} + J_{21})\sin a_{AB}\cos a_{AB} \tag{4}$$

$$\delta a_{AB} \approx J_{12}\cos^2 a_{AB} - J_{21}\sin^2 a_{AB} + (J_{11} - J_{22})\sin a_{AB}\cos a_{AB} \tag{5}$$

For an angle $\theta_{ABC} = a_{AC} - a_{AB}$ the variation becomes to the same approximation

$$\delta\theta_{ABC} \approx (J_{12} + J_{21})(\sin^2 a_{AB} - \sin^2 a_{AC}) + \frac{1}{2}(J_{11} - J_{22})(\sin 2a_{AC} - \sin 2a_{AB}). \tag{6}$$

Replacing the elements of $\mathbf{J}$ with the strain parameters according to (2) we arrive at the equivalent relations

$$\delta a_{AB} \approx \frac{\gamma_2}{2}\cos 2a_{AB} + \omega + \frac{\gamma_1}{2}\sin 2a_{AB}, \tag{7}$$

$$\frac{\delta s_{AB}}{s_{AB}} \approx \frac{\Delta}{2} - \frac{\gamma_1}{2}\cos 2a_{AB} + \frac{\gamma_2}{2}\sin 2a_{AB}, \tag{8}$$

$$\delta\theta_{ABC} \approx \frac{\gamma_2}{2}(\cos 2a_{AC} - \cos 2a_{AB}) + \frac{\gamma_1}{2}(\sin 2a_{AC} - \sin 2a_{AB}). \tag{9}$$

From the above relations it is obvious that dilatation Δ can be determined only when also distances are observed, while the determination of the rotation ω requires the inclusion of azimuth observations.

In the method of **Frank** (1966) two equations of the form of (9) are formulated for the two independent angles of a triangle and are uniquely solved for the two constant shear strain components γ_1, γ_2 within the triangle. Note that Frank's method presupposes that the triangle angles are not the raw observed ones but have been corrected so that they sum up to 180° at each epoch, so that the 3 variations satisfy the condition $\delta\theta_{ABC} + \delta\theta_{BCA} + \delta\theta_{CAB} = 0$. **Bibby** (1975), claims that a graphical method equivalent to that of Frank had been already used by **Wellman** (1955). The method is subsequently used by **Pope**, **Stearns** & **Whitten** (1966), **Thatcher** (1975), **Dunbar**, **Boore** & **Thatcher** (1980), **Wellman** (1981), **Jouanne**, **Menard** & **Jault** (1994), **Jouanne**, **Hippolyte**, **Gamond** & **Martinod** (2001).

Savage & **Burford** (1970) extended the approach of **Frank** (1966) from triangles to quadrilaterals. The 8 variations of the 8 quadrilateral angles as observed in the two epochs without any correction are expressed in terms of the 2 shear components and a least squares solution for γ_1 and γ_2 is obtained. Similar extensions with least squares fits have been used by **Wellman** (1981), **Hunstad**, **Selvaggi**, **D'Agostino**, **England**, **Clarke** & **Pierozzi** (2003).

Savage & Prescott (1973), following earlier original work of **Hofmann** (1968), (see also **Scholz** & **Fitch**, 1969) utilized eq. (4) which with $J_{11} = \varepsilon_{11}$, $J_{22} = \varepsilon_{22}$, $J_{12} = \varepsilon_{12} + \omega$, $J_{21} = \varepsilon_{21} - \omega$ is converted to

$$\frac{\delta s_{AB}}{s_{AB}} \approx \sin^2 a_{AB}\varepsilon_{11} + \cos^2 a_{AB}\varepsilon_{22} + \sin 2a_{AB}\varepsilon_{12} \tag{10}$$

for the observed 15 lines of a 7-point trilateration network, to obtain estimates of the 3 strain components ε_{11}, ε_{22}, ε_{12}, assumed constant over the whole network through a least squares fit. The same method is subsequently used by **Savage** & **Burford** (1973), **Savage** & **Prescott** (1976), **Savage**, **Prescott**, **Lisowski** & **King** (1981), **Prescott** & **Savage** (1976), **Savage**, **Prescott**, **Lisowski** & **King** (1978), **Savage**, **Lisowski** & **Prescott** (1981a), **Savage**, **Lisowski** & **Prescott** (1981b), **Prescott**, **Lisowski** & **Savage** (1981).

Prescott (1976) modified Frank's method by switching from angles

$\theta_{ABC} = a_{AC} - a_{AB}$ to the directly observed directions $\phi_{AB} = a_{AB} - a_A$. The corresponding variations $\delta\phi_{AB} = \delta a_{AB} - \delta a_A$ bring in an additional parameter δa_A for each observing station A. With δa_{AB} expressed as in eq. (7) it hold that

$$\delta\phi_{AB} = \delta a_{AB} - \delta a_A = \frac{\gamma_2}{2}\cos 2a_{AB} + \frac{\gamma_1}{2}\sin 2a_{AB} - (\delta a_A - \omega) \tag{11}$$

and a least squares solution is possible for γ_1, γ_2 and the set of per station "nuisance" parameters $\delta\tilde{a}_A = \delta a_A - \omega$, in which the common rotation ω is absorbed. The method is used subsequently by **Prescott**, **Savage** & **Kinoshita** (1979), **Prescott** & **Savage** (1976), **Snay** (1986), **Sauber** (1989), **Davies** (1996), **Davies**, **England**, **Parsons**, **Billiris**, **Paradissis** & **Veis** (1997), **Bawden**, **Donnellan**, **Kellogg**, **Dong** & **Rundle** (1997).

The next greatly influential work, based also on the assumption of constant stains within a region, is the "simultaneous adjustment method" of **Bibby** (1981), which is based on previous work (**Bibby** 1973, 1975, 1976) and it is further elaborated in **Bibby** (1982). The same method applied to the surface of the earth-sphere rather than the horizontal plane has been presented independently by **Snay** & **Cline** (1980). The basic idea is to adjust simultaneously observations carried out at different epochs based on the fundamental relation for deformation between an initial reference epoch t_0 and the observation epoch t

$$\mathbf{x}(t) = \mathbf{F}(t)\mathbf{x}(t_0) \equiv \mathbf{F}(t)\mathbf{x}_0 = \mathbf{x}_0 + \mathbf{J}(t)\mathbf{x}_0 = \mathbf{x}_0 + (t - t_0)\dot{\mathbf{J}}\mathbf{x}_0 \tag{12}$$

where in relation to the general relation $\mathbf{x}(t) = \mathbf{F}\left(\mathbf{x}(t_0), t\right)\mathbf{x}(t_0) + \mathbf{c}(t) = = \mathbf{x}(t_0) + \mathbf{J}\left(\mathbf{x}(t_0), t\right)\mathbf{x}(t_0) + \mathbf{c}(t)$, it has been assumed that: (a) $\mathbf{J}\left(\mathbf{x}(t_0), t\right) = \mathbf{J}(t)$, i.e. constant strains over the whole area for every epoch t, (b) $\mathbf{J}(t) = (t - t_0)\dot{\mathbf{J}}$, i.e. strains linear in time with $\mathbf{J}(t_0) = \mathbf{0}$ and (c) $\mathbf{c}(t) = \mathbf{0}$ has been conveniently set for the temporal displacement of the area as a whole, which is in any case undetectable by geodetic means. Since any geodetic observable involves 2 or 3 points its can be expressed as a function of their coordinates $\mathbf{x}(t)$ at the observation epoch t. Using eq. (12), $\mathbf{x}(t) = \mathbf{x}_0 + (t - t_0)\dot{\mathbf{J}}\mathbf{x}_0$, the observables become functions of the initial coordinates $\mathbf{x}_0$ and the strain rate parameters $\dot{\mathbf{J}}$, which utilizing the relation resulting from the time differentiation of eq. (2) may be conveniently replaced by another set of strain rate parameters such as $\dot{\varepsilon}_{11}$, $\dot{\varepsilon}_{12}$, $\dot{\varepsilon}_{22}$, $\dot{\omega}$, or $\dot{\Delta}$, $\dot{\gamma}_1$, $\dot{\gamma}_2$, $\dot{\omega}$. Bibby's method found further application either through the use of the DYNAP software of **Drew** & **Snay** (1989), implemented by **Feigl**, **King** & **Jordan** (1990), **Snay** & **Matsikari** (1991), **Liu**, **Zoback** & **Segall** (1992), or independently by **Walcott** (1984), **Walcott** (1987), **Navarro**, **Catalao**, **Miranda**, & **Fernandes** (2003). **Reilly** & **Gubler** (1990), extended the method to the case of non-constant strain

rates varying linearly within the region of study. The exposition by **Feigl**, **King** & **Jordan** (1990), appears to have been quite successful since it draw the attention of several researchers to the approach, e.g. **Jouanne**, **Menard** & **Jault** (1994), **Walpersdorf**, **Vigny**, **Manurung**, **Subarya** & **Sutisna** (1998), **Ferhat**, **Feigl**, **Ritz** & **Souriau** (1998), **Duong** & **Feigl** (1999), **Sue** et al. (2000), **Navarro**, **Catalao**, **Miranda** & **Fernandes** (2003), **Vigny** et al. (2003), **Serpelloni**, **Anzidei**, **Baldi**, **Casula** & **Galvani** (2005).

3. Linear displacement model by least-squares fit

In the methods developed by **Frank** (1966) (modified by **Savage** & **Prescott**, 1973, and **Prescott**, 1976) and especially **Bibby** (1981), the crucial point is the relation of observables to a set of strain parameters or just those of their combinations that can be estimated by the data at hand. This relation is based on the fundamental assumption that $\mathbf{x}(t) = \mathbf{F}(t)\mathbf{x}_0 + \mathbf{c}(t) = \mathbf{x}_0 + \mathbf{J}(t)\mathbf{x}_0 + \mathbf{c}(t)$ so that displacements $\mathbf{u}(t) = \mathbf{x}(t) - \mathbf{x}(t_0)$ are linear functions $\mathbf{u}(t) = \mathbf{J}(t)\mathbf{x}_0 + \mathbf{c}(t) - \mathbf{c}_0 = \mathbf{J}(t)\mathbf{x}_0 + \delta\mathbf{c}(t)$ of the initial epoch coordinates $\mathbf{x}_0 = \mathbf{x}(t_0)$, or under the assumption that strain parameters change linearly with time, $\mathbf{u}(t) = (t - t_0)\dot{\mathbf{J}}\mathbf{x}_0 + (t - t_0)\dot{\mathbf{c}}$. The last model is more appropriate for multi-epoch observations, while both models are equivalent when only two observation epochs are at hand. These assumptions may hold for the whole network if it small, or separately within each subnetwork resulting from a tectonically meaningful division of the region, with each network triangle treated as a separate sub-region in the extreme case. A different matrix $\mathbf{J}_K = \mathbf{J}(\mathbf{s}_K)$ or $\dot{\mathbf{J}}_K = \dot{\mathbf{J}}(\mathbf{s}_K)$ is assigned to each sub-region K with its own strain parameters $\mathbf{s}_K$ or strain rates $\dot{\mathbf{s}}_K$, which are the unknowns to be estimated in a least squares adjustment. When identical observations are carried out in each epoch the (linearized) observations $\mathbf{b}(t) = \mathbf{A}\mathbf{x}(t)$ share the same design matrix $\mathbf{A}$ and can be differenced to obtain models where only displacement appear $\delta\mathbf{b}(t) \equiv \mathbf{b}(t) - \mathbf{b}(t_0) = \mathbf{A}[\mathbf{x}(t) - \mathbf{x}(t_0)] = \mathbf{A}\mathbf{u}(t)$. Expressing the displacement $\mathbf{u}_i$ of each point P_i in terms of the strain parameters of its proper sub-region K, we arrive at linear models

$$\mathbf{u}_i = \mathbf{J}(\mathbf{s}_K)\mathbf{x}_{i0} + \delta\mathbf{c}_K = \mathbf{C}(\mathbf{x}_{0i})\mathbf{s}_K + \delta\mathbf{c}_K \approx \mathbf{C}(\mathbf{x}_i^{\mathrm{ap}})\mathbf{s}_K + \delta\mathbf{c}_K = \mathbf{C}_i\mathbf{s}_K + \delta\mathbf{c}_K$$

or

$$\mathbf{u}_i(t) = (t - t_0)\left[\dot{\mathbf{J}}(\dot{\mathbf{s}}_K)\mathbf{x}_{0i} + \dot{\mathbf{c}}_K\right] = (t - t_0)\left[\mathbf{C}(\mathbf{x}_{0i})\dot{\mathbf{s}}_K + \dot{\mathbf{c}}_K\right] \approx$$
$$\approx (t - t_0)\left[\mathbf{C}(\mathbf{x}_i^{\mathrm{ap}})\dot{\mathbf{s}}_K + \dot{\mathbf{c}}_K\right] = (t - t_0)\left[\mathbf{C}_i\dot{\mathbf{s}}_K + \dot{\mathbf{c}}_K\right]$$

where initial coordinates $\mathbf{x}_{0i}$ have been replaced by the common for all epochs approximate values $\mathbf{x}_i^{\mathrm{ap}}$. For all the displacements we have the linear model

$\mathbf{u} = \mathbf{C}\mathbf{s} + \mathbf{D}\mathbf{c}$, or $\mathbf{u}(t) = \mathbf{C}(t)\dot{\mathbf{s}} + \mathbf{D}(t)\dot{\mathbf{c}}$, where $\mathbf{s}$ or $\dot{\mathbf{s}}$ are the strain parameters or their rates for all the sub-networks and $\mathbf{c}$ or $\dot{\mathbf{c}}$ are the relative displacements or velocities of the sub-regions. Finally the adjustment model $\delta\mathbf{b}(t) = \mathbf{A}\mathbf{u}(t)$ for 2 epochs or $\delta\mathbf{b}(t_j) = \mathbf{A}\mathbf{u}(t_j)$ for multiple epochs t_i, taking into consideration the observation errors $\mathbf{v}$ takes the form $\delta\mathbf{b} = (\mathbf{AC})\mathbf{s} + (\mathbf{AD})\mathbf{c} + \mathbf{v}$ for two epochs or $\delta\mathbf{b}(t_j) = [\mathbf{AC}(t_j)]\dot{\mathbf{s}} + [\mathbf{AD}(t_j)]\dot{\mathbf{c}}$, $j = 1,2,...,T$, for multiple epochs. From the adjustment of the observation differences estimates of the strain parameters are derived as well as the relative displacements or relative velocities between sub-regions (no displacement or no velocity must be assumed for one sub-region, since a common rigid motion of the whole network cannot be detected). In many cases it is not possible to have exactly the same networks and/or observations in all epochs, since monuments may be destroyed or new monuments constructed. In such a case the design matrix $\mathbf{A}(t_j)$ is different at each epoch and the initial adjustment model $\mathbf{b}(t_j) = \mathbf{A}_j\mathbf{x}(t_j) = \mathbf{A}_j\mathbf{x}_{0j} + \mathbf{A}_j\mathbf{u}(t_j)$ contains in addition to the displacements the initial coordinates $\mathbf{x}_{0j}$, so that the initial coordinates $\mathbf{x}_0$ of all epoch networks appear as additional parameters in the adjustments together with the strain and strain rate parameters.

An alternative to the above "raw observation" approach is the "displacement" approach, where the network is adjusted for all epoch observations using initial coordinates and per epoch displacements or velocities as unknown parameters. Although the degrees of freedom are less than in the "raw observation" approach, the "displacement" method has the advantage that the a posteriori least-squares fitting of strain (or strain rate) parameters to the displacements of each sub-region produces residuals which give an indication of the success of the fit of linear model in each separate sub-region and may lead to the unification or further division of the sub-regions until small residuals or appropriate statistical tests indicate a satisfactory model choice. A special case of this approach is the previously described finite element method where the strain parameters are uniquely derived by displacements at each triangular sub-region without need for a least-squares fit.

Such approaches with different strain (or strain rate) parameters for the whole region or different sub-regions or even triangular sub-regions have been used by many authors, e.g., **Frank** (1966), **Scholz** & **Fitch** (1969), **Savage** & **Burford** (1970), **Bibby** (1973), **Savage** & **Burford** (1973), **Savage** & **Prescott** (1973), **Bibby** (1975), **Thatcher** (1975), **Prescott** (1976), **Savage** & **Prescott** (1976), **Bibby** & **Walcott** (1977), **Savage**, **Prescott**, **Lisowski** & **King** (1978), **Fujii** & **Nakane** (1979), **Prescott**, **Savage** & **Kinoshita** (1979), **Slawson** & **Savage** (1979), **Dunbar**, **Boore** & **Thatcher** (1980), **Bibby** (1981), **Prescott** (1981), **Savage**, **Lisowski** & **Prescott** (1981a), **Savage**, **Lisowski** & **Prescott** (1981b), **Savage**, **Prescott**, **Lisowski** & **King** (1981), **Bibby** (1982), **Lambeck** & **Coleman** (1984), **Snay** (1986), **Kasser** et al. (1987), **Drew** & **Snay** (1989), **Feigl**, **King** & **Jordan**

(1990), **Grant** (1990), **Reilly** & **Gubler** (1990), **Darby** & **Williams** (1991), **Snay** & **Matsikari** (1991), **Liu**, **Zoback** & **Segall** (1992), **Reilly** et al. (1992), **Deniz** et al. (1993), **Dong** (1993), **Donnellan**, **Hager**, **King** & **Herring** (1993), **Stiros** (1993), **Fujii** (1995), **Davies** (1996), **Bawden**, **Donnellan**, **Kellogg**, **Dong** & **Rundle** (1997), **Davies**, **England**, **Parsons**, **Billiris**, **Paradissis** & **Veis** (1997), **Dong**, **Herring** & **King** (1998), **Duong** & **Feigl** (1999), **Calais** et al. (2000), **Anzidei** et al. (2001), **Gan** & **Prescott** (2001), **Hamdy** (2001), **Jouanne**, **Hippolyte**, **Gamond** & **Martinod** (2001), **Savage**, **Svarc** & **Prescott** (2001), **Hunstad**, **Selvaggi**, **D'Agostino**, **England**, **Clarke** & **Pierozzi** (2003), **Khazaradze** & **Klotz** (2003), **Navarro**, **Catalão**, **Miranda** & **Fernandes** (2003), **Ruiz** et al. (2003), **Vigny** et al. (2003), **Savage**, **Gan**, **Prescott** & **Svarc** (2004), **Savage**, **Svarc** & **Prescott** (2004), **Hamdy**, **Park** & **Lim** (2005), **Serpelloni**, **Anzidei**, **Baldi**, **Casula** & **Galvani** (2005), **Delikaraoglou**, **Billiris**, **Paradissis**, **England**, **Parsons** & **Clarke** (2006), **Kotzev**, **Nakov**, **Georgiev**, **Burchfiel** & **King** (2006), **Palano** et al. (2006), **Walpersdorf** et al. (2006), **Simons** et al. (2007), **Reddy** & **Sunil** (2008).

A shortcoming of the linear model is its inability to take into account the interpolation error and the method works well only when strain parameters vary little between adjacent sub-regions. In contrast to the finite element method with triangular elements, the existence of the interpolation is manifested in the accuracy of the results though not in the proper way. Indeed when the linear model fails to fit the local behaviour of the displacements the estimated observation residuals become large and the estimated covariances are inflated by a large a-posteriori variance $\hat{\sigma}^2$. However these accuracy estimates are based on the assumption that the observation errors are a zero-mean white noise, while the true errors are affected by the strongly correlated displacements in neighbouring points. This spatial correlation is properly taken into account in the collocation method for the interpolation of displacements or velocities (paragraph 6).

4. The problem of datum definition and invariance of strain parameters

In deformation studies it is necessary to differentiate between "absolute" networks where some points are remaining fixed in time and "relative" networks where all points are moving (**Chrzanowski**, **Chen** & **Secord**, 1983). Of course the eventual increase in observational accuracy has limited the relevance of absolute networks to engineering level studies only while, even global networks are relative ones. In any case, the choice of reference system affects the computed displacements and the non-invariant of the strain parameters. In absolute networks the reference system must be chosen once for the sub-network of fixed points. In relative networks a reference system must be chosen for every observation epoch. Among the strain parameters some are independent while others are affected by the choice of the reference system, always in relation to the type of available observations.

The principal strains $e_{\max}$, $e_{\min}$ and maximum shear strain γ are always invariant. Dilatation Δ is an invariant only when distance observations have been performed in both epochs with the same unit of distance (through calibration of the relevant instruments) otherwise it is affected by the choice of scale at the epochs where only angles are observed. The rotation ω depends by the choice of orientation in both observation epochs, except when azimuths have been observed in both epochs. However the accuracy of azimuths is so relatively low that invariance of ω cannot be practically claimed. In any case the effect is a common constant added to all rotations so that differences (relative rotations) between different sub-regions (or points when displacements are interpolated) remain invariant. The choice of origin for both epochs adds a constant translation to all sub-regions so it does not affect relative translations. When distances are observed relative translation vectors maintain their magnitudes but their orientation changes by an angle common to all.

The first study for the invariance of strain parameters as derived in the infinitesimal approach was given by **Dermanis** (1981). His results show that the effect of reference system choice is somewhat different from what was expected in view of the tensorial character of the strain tensor as a result of the approximations. **Van Mierlo** (1982) studied the related problem of the estimability of strain parameters, determining which of their linear combinations can be estimated for different observation types. **Dermanis** (1985), gave criteria of invariance in the case where a method of displacement interpolation is used for the derivation of strain parameters. **Dermanis** & **Grafarend** (1992), studied the invariance of strain parameters without the approximations of the infinitesimal approach in both the horizontal and three-dimensional case. **Xu**, **Shimada**, **Fujii** & **Tanaka** (2000), studied the estimability of both strain parameters and three-dimensional relative translations.

In the older days the reference system was usually introduced in horizontal networks by fixing the coordinates of one point and one azimuth (minimal constraints) and the datum defect problem was resolved in deformation studies by fixing the same point and azimuth in both epochs. **Meissl** (1962, 1969) introduced a different approach to the single epoch datum problem. He proposed a best fitting of the network to the imaginary network formed by a set of approximate coordinates, in which case the reference system of the approximate network passes over to the real one. From the algebraic point of view the best fitting criterion $\mathbf{x}^T\mathbf{x} = \sum_i \mathbf{x}_i^T\mathbf{x}_i = = \min$, where $\mathbf{x}_i$ are the corrections to the approximate coordinates $\mathbf{x}_i^0$ of station i, is equivalent to a set of "inner constraints" $\mathbf{E}^T\mathbf{x} = \sum_i \mathbf{E}_i^T\mathbf{x}_i = \mathbf{0}$, with

$$\mathbf{E}_i = \begin{bmatrix} 1 & 0 & -y_i^0 & x_i^0 \\ 0 & 1 & x_i^0 & y_i^0 \end{bmatrix}, \qquad \mathbf{E}_i = \begin{bmatrix} 1 & 0 & 0 & 0 & z_i^0 & -y_i^0 & x_i^0 \\ 0 & 1 & 0 & -z_i^0 & 0 & x_i^0 & y_i^0 \\ 0 & 0 & 1 & y_i^0 & -x_i^0 & 0 & z_i^0 \end{bmatrix} \tag{13}$$

for the 2- and 3-dimensional case respectively. For horizontal networks the first 2 columns of $\mathbf{E}_i$ take care of the datum origin defect, the 3rd of the orientation and the 4th of the scale. For 3-dimensional networks the first 3 columns of $\mathbf{E}_i$ take care of the datum origin defect the columns 4-5-6 of the orientation and the 7th of the scale. Thus the last column of $\mathbf{E}_i$ should be removed when distances are observed. The application of the inner constraints $\mathbf{E}^T\mathbf{x}=\mathbf{0}$ in the adjustment of the observation equations $\mathbf{b}=\mathbf{A}\mathbf{x}+\mathbf{v}$, $\mathbf{v}\sim(\mathbf{0},\sigma^2\mathbf{P}^{-1})$, produces the "inner coordinates" $\hat{\mathbf{x}}_+$ which are the same as those produced by taking the pseudoinverse of the singular normal equations matrix $\mathbf{x}_+=(\mathbf{A}^T\mathbf{P}\mathbf{A})^+\mathbf{A}^T\mathbf{P}\mathbf{b}$. The inner coordinates can be also computed by an easy to perform transformation of any coordinate solution $\mathbf{x}$ produced by any minimal constraints, namely

$$\mathbf{x}_+=\mathbf{H}\mathbf{x}=[\mathbf{I}-\mathbf{E}(\mathbf{E}^T\mathbf{E})^{-1}\mathbf{E}^T]\mathbf{x}\,, \tag{14}$$

thus avoiding the explicit or implicit use of the pseudoinverse.

The first to apply the ideas of Meissl to deformation studies has been **Brunner** (1979), for 3-dimensional networks, while the case of horizontal networks has been treated in **Brunner**, **Coleman** & **Hirsch** (1981) (see also **Margrave** & **Nyland**, 1980). When displacements $\mathbf{u}_+=\mathbf{x}'_+-\mathbf{x}_+$ are produced by differences of inner coordinates they are minimized (in the sense that $\mathbf{u}^T\mathbf{u}=\min$) and the effect of datum defects on strain parameters is filtered out. At first sight it is not obvious at all why the best fitting of both epoch networks to a common approximate network guarantees the best fitting to each other and rigorously speaking this is not the case. It can be proved however that within the linearization approximation for small rotation angles and scale factor close to one, $\mathbf{u}_+$ is the same as the one obtained by $\mathbf{u}_+=\tilde{\mathbf{x}}'-\mathbf{x}$, where $\tilde{\mathbf{x}}'=\mathbf{x}'-\mathbf{E}(\mathbf{E}^T\mathbf{E})^{-1}\mathbf{E}^T(\mathbf{x}'-\mathbf{x})$ results from the coordinate transformation of $\mathbf{x}'$ which best fits the first epoch coordinates $\mathbf{x}$. This means that $\mathbf{u}_+$ can be obtained by directly applying the same transformation $\mathbf{u}_+=\mathbf{H}\mathbf{u}$ to the displacements $\mathbf{u}=\mathbf{x}'-\mathbf{x}$ resulting from any minimally constrained solutions. An alternative procedure is to use the coordinates $\mathbf{x}$ of the first epoch adjustment produced by any minimal constraints as approximate coordinates for the second epoch so that $\mathbf{x}'=\mathbf{u}$ and the inner coordinates $\mathbf{x}'_+=\mathbf{u}_+$ satisfy $\mathbf{u}^T\mathbf{u}=\min$. If a rigid (or similarity) transformation can be applied to the second epoch coordinates $\mathbf{x}'$ to best fit them to those of the first epoch $\mathbf{x}$, either by a rigorous non-linear solution or by an iterative one through linearization, even smaller displacements can be produced.

The first two inner constraints have the form $x_C\equiv\frac{1}{N}\sum_i x_i=\frac{1}{N}\sum_i x_i^0\equiv x_C^0$, $y_C\equiv\frac{1}{N}\sum_i y_i=\frac{1}{N}\sum_i y_i^0\equiv y_C^0$, which means that the network barycenter is main-

tained. The third can be written $(1/D)\sum_i(-y_i^0\delta x_i + x_i^0\delta y_i) = 0$ where $D^2 = \sum_i(x_i^2 + y_i^2)$, meaning that displacements are minimized in all directions. **Prescott** (1981), introduced the idea of "outer coordinates", which result by maintaining the first two "translation" constraints and replacing the third "orientation" constraint with $(1/E)\sum_i y_i^0\delta x_i = 0$, where $E^2 = \sum_i y_i^2$ so that only displacements along the x-axis are minimized, while displacements in the direction of the y-axis are free to become larger. In situations where it is a priori known that displacements should occur in the direction of the fault, the outer coordinates can be used by rotating the reference system so that the y-axis takes the direction of the fault. The method, which met some popularity, lacks serious theoretical but also operational foundation since it fails to distinguish between relative motion and deformation, as already done by **Whitten** (1960) and **Whitten & Claire**, 1960 in similar situations (see paragraph 8).

A more reasonable proposal have been the "special stations solution" of **Gu** & **Prescott** (1986), see also **Segall** & **Mathews** (1988), where not all but only the displacements of a selected set of "fixed" stations are minimized. Similar ideas however for such "partial inner constraint" solutions, including appropriate statistical tests, had already been introduced in the geodetic literature, see e.g. **Koch** & **Fritsch** (1981), **Niemeier** (1981).

When multiple epochs are involved, e.g. in coordinate series from GPS permanent stations, in principle a different reference system must be defined for every epoch. However the coordinates per epoch are not independent since they must vary slowly and in a smooth way except for episodic changes associated with earthquakes, landslides, antenna changes, etc. Thus the evolution of coordinates must be modeled by either deterministic or stochastic tools (**Dermanis** & **Rossikopoulos**, 1988). The simplest model, already applied for two epoch studies is the linear evolution in time $\mathbf{x}_t = \mathbf{x}_0 + (t - t_0)\mathbf{v}$, involving initial coordinates $\mathbf{x}_0 = \mathbf{x}(t_0)$ and velocities $\mathbf{v}$. In this case the displacement $\mathbf{u}_t = (t - t_0)\mathbf{v}$ and the displacement gradient $\mathbf{J}(t) = \partial\mathbf{u}_t / \partial\mathbf{x}_0 = (t - t_0)[\partial\mathbf{v} / \partial\mathbf{x}_0]$ are replaced by the velocity $\mathbf{v}$ and the velocity gradient $\mathbf{L} = \partial\mathbf{v} / \partial\mathbf{x}_0 = \dot{\mathbf{J}}$. The problem is that a time-dependent coordinate transformation, e.g. $\tilde{\mathbf{x}}_t = \mathbf{R}_t\mathbf{x}_t + \mathbf{d}_t$ fails to reserve the linear in time character of the untransformed model. For this reason only model-preserving transformations are allowed for each independent tectonic block such as a rigid motion $\tilde{\mathbf{x}}_t = \mathbf{R}\mathbf{x}_t + \mathbf{d}$ in the plane ($\mathbf{R}$ and $\mathbf{d}$ constant) or a rigid rotation around a fixed pole with constant angular velocity. Such transformations involve parameters which must be determined in a way that the reduced velocities $\tilde{\mathbf{v}}$ are minimized in a least-squares sense. More details are given in paragraph 8.

5. Alternatives to the linear model for displacements

Several authors have tried to go beyond the linear model by interpolating displacements $\mathbf{u}$ or velocities $\mathbf{v}$ for the computation of strains or strain rates, introducing models of the form $\mathbf{u}(\mathbf{x}) = \mathbf{u}(\mathbf{x}, \mathbf{a})$ with $\mathbf{a}$ being unknown coefficients to be estimated from the available data $\mathbf{u}_i = \mathbf{u}(\mathbf{x}_i)$. The most obvious choice for each displacement component is the use of polynomials or more generally models of the form $f(P) = \sum_{k=1}^{m} a_k \varphi_k(P) = \boldsymbol{\varphi}_P^T \mathbf{a}$ with $(\boldsymbol{\varphi}_P)_k = \varphi_k(P)$, where $\varphi_k(P)$ are known "base" functions. Taking into account the observational errors e_i the observation model becomes $f_i = f(P_i) + e_i = \sum_{k=1}^{m} a_k \varphi_k(P_i) + e_i$, $i = 1,...,n$, $m < n$, or $\mathbf{f} = \boldsymbol{\Phi}\mathbf{a} + \mathbf{e}$ with $\Phi_{ik} = \varphi_k(P_i)$. From a least squares fitting $\mathbf{e}^T \mathbf{P} \mathbf{e} = \min$ the coefficient estimates and the interpolating function are obtained

$$\hat{\mathbf{a}} = (\boldsymbol{\Phi}^T \mathbf{P} \boldsymbol{\Phi}) \boldsymbol{\Phi}^T \mathbf{P} \mathbf{f}, \qquad \hat{f}(P) = \boldsymbol{\varphi}_P^T \hat{\mathbf{a}} = \boldsymbol{\varphi}_P^T (\boldsymbol{\Phi}^T \mathbf{P} \boldsymbol{\Phi}) \boldsymbol{\Phi}^T \mathbf{P} \mathbf{f}. \tag{15}$$

Polynomial models have been used by **Haines** (1982), though not for strain determination but for the interpolation of strain rates in order to obtain vector fields through integration. The same method has been applied by **Walcott** (1984). **Chrzanowski**, **Chen**, & **Secord** (1983) were the first to propose polynomial models for displacements though they used only the linear part in their applications. Polynomial models have been used e.g. by **Kato** & **Nakajima** (1989), **Darby** & **Williams** (1991), **Clarke** (1996). Legendre and Hermite polynomials have been used by **Asteriadis**, **Schwan** & **Arabelos** (1988).
Chebychev polynomials have been used by **Sato**, **Miura** & **Tachibana** (1993).

Haines & **Holt** (1993) introduced a method for the determination of a continuous velocity field from strain rates obtained from the summation of earthquake moment tensors, which was later modified for the interpolation of velocities and the determination strain rates. Since a rigid rotation on the sphere with rotation vector $\boldsymbol{\omega}$ results in a velocity field $\mathbf{v}(\mathbf{x}) = [\boldsymbol{\omega}\times]\mathbf{x}$, for a general field $\mathbf{v}(\mathbf{x})$ resulting from a non-rigid deformation there exists a unique vector $\boldsymbol{\omega}(\mathbf{x})$ void of any precise physical meaning (points do not rotate!) such that $\mathbf{v}(\mathbf{x}) = [\boldsymbol{\omega}(\mathbf{x})\times]\mathbf{x} = -[\mathbf{x}\times]\boldsymbol{\omega}(\mathbf{x})$. They modeled the 3 components of $\boldsymbol{\omega}(\mathbf{x})$ as truncated polynomials in powers of the 3 components of $\mathbf{x}$. **Wu**, **Shuai**, **Wang**, **Zhou** & **Li** (1997) modified the method for the interpolation of GPS derived velocities and strain computation, using the same polynomial modeling of $\boldsymbol{\omega}(\mathbf{x}) = \boldsymbol{\omega}(\mathbf{x}, \mathbf{a}) = \mathbf{B}(\mathbf{x})\mathbf{a}$, where $\mathbf{a}$ contains the unknown polynomial coefficients. Taking observation errors $\mathbf{e}_i$ the observed velocities at each point i become $\mathbf{v}_i = -[\mathbf{x}_i\times]\boldsymbol{\omega}(\mathbf{x}_i) + \mathbf{e}_i = -[\mathbf{x}_i\times]\mathbf{B}(\mathbf{x}_i)\mathbf{a} + \mathbf{e}_i \equiv$ $\equiv \mathbf{A}_i\mathbf{a} + \mathbf{e}_i$ and a least squares fit gives the coefficient estimates $\hat{\mathbf{a}}$, which provide the interpolated field $\hat{\mathbf{v}}(\mathbf{x}) = -[\mathbf{x}\times]\mathbf{B}(\mathbf{x})\hat{\mathbf{a}}$. Of course the method is equivalent to a certain type of direct modeling of $\mathbf{v}(\mathbf{x})$ by polynomials, since if the vector $\mathbf{B}(\mathbf{x})\mathbf{a}$ has polynomial components so does $\mathbf{A}(\mathbf{x})\mathbf{a} = -[\mathbf{x}\times]\mathbf{B}(\mathbf{x})\mathbf{a}$. The only advantage is

that the zero order term coefficients $\boldsymbol{\alpha}_0$ in $\boldsymbol{\omega}(\mathbf{x})$ give the common rotational rigid motion of a sub-region with rotation vector $\boldsymbol{\omega} = \boldsymbol{\alpha}_0$ while the remaining coefficients describe the deformation within the sub-region. **Haines**, **Jackson**, **Holt** & **Agnew** (1998) and **Kreemer**, **Haines**, **Holt**, **Blewitt** & **Lavallee** (2000), addressed the same problem using instead of polynomials bicubic splines on a curvilinear grid.

A related method of interpolation is that of piece-wise polynomials or splines with smoothness conditions imposed at the boundaries of their domains. Spline interpolation has been used by **Altiner** (2001) for velocities and **Kreemer**, **Holt** & **Haines** (2003) who used bi-cubic Bessel splines for the interpolation of velocities and model strain rates. The "splines in tension" method of **Wessel** & **Bercovici** (1998), has been used for the interpolation of velocities by **Jin** & **Park** (2006) and **Gan** et al. (2007).

Another type of interpolation methods are the "moving" or "localized" interpolations which are applied separately at any point of interest P (moving) and treat the available data at points P_i in relation to their relative positions with respect to P (localized). **Shen**, **Jackson** & **Ge**, (1996) use such a method where (separately for every point P) strain rates corresponding to a spatially linear velocity field are fitted to the observed velocities with an additional modification of their covariances. All covariances between velocity components at data points P_i and P_k are multiplied by a factor $\lambda_{ik} = \mathrm{e}^{d_i^2/\sigma_D^2}\, e^{d_k^2/\sigma_D^2}$ depending on their distances $d_i = d_{PP_i}$, $d_k = d_{PP_k}$ from P, σ_D being a constant. The locally applied method is equivalent to replacing the observed velocities $\mathbf{v}_i$ with their "shrunk" versions $\lambda_i \mathbf{v}_i = \mathrm{e}^{d_i^2/\sigma_D^2}\, \mathbf{v}_i$ ($\lambda_{ik} = \lambda_i \lambda_k$) and modifying their covariances accordingly. Since $\lambda_i \to 0$ as $d_i = d_{PP_i} \to \infty$, the solution depends more on the velocities of the neighbouring points and less on the more distant ones. The method has also been used by **Sagiya**, **Miyazaki** & **Tada** (2000), **Sagiya** (2004), **Sagiya**, **Nishimura** & **Iio** (2004) and **D'Agostino** & **Selvaggi** (2004). A moving-localized interpolation method has also been used by **Clarke** et al. (1998), who estimated velocity gradients separately for each point on a grid by using only observed velocities within a fixed distance, provided that there exist at least 4 such points.

Some authors take advantage of the free software for interpolation on a grid, and derive strains from the numerical differentiation of the grid velocity values. The GMT (General Mapping Tools) of **Wessel** & **Smith** (1991, 1995) has been used e.g. by **Miura**, **Sato**, **Tachibana**, **Satake** & **Hasegawa** (2002), **Miura**, **Sato**, **Hasegawa**, **Suwa**, **Tachibana** & **Yui** (2004), **Takayama** & **Yoshida** (2007).

An interesting alternative to classical deterministic and stochastic models for deriving the velocity field from discrete data is the robust smoothing and exploratory data analysis (EDA) of **Toya** & **Kasahara** (2005).

6. Strain parameters from minimum mean square error prediction (collocation)

A method to interpolate the 2 components of discrete displacements $\mathbf{u}_i = \mathbf{u}(P_i)$ available at discrete points P_i, $i = 1,...,n$, is through the use of minimum norm interpolation utilizing a model where a function is modeled as a linear combination $f(P) = \sum_{k=1}^{m} a_k \psi_k(P)$ of known base functions $\psi_k(P)$, $k = 1,...,m$, with $m > n$. For the available data the equations $f_i \equiv f(P_i) = \sum_{k=1}^{m} a_k \psi_k(P_i)$, which can be written in the matrix form $\mathbf{f} = \mathbf{F}\mathbf{a}$ with $F_{ik} = \psi_k(P_i)$, have infinite number of solutions. A unique one is obtained by minimizing the norm $\|\mathbf{a}\|^2 = \mathbf{a}^T \mathbf{W} \mathbf{a}$, where the "weight" matrix $\mathbf{W}$ is a symmetric positive definite matrix, which can be taken to be diagonal without loss of generality (if not, a change to base functions with diagonal weight matrix can be applied by taking appropriate linear combinations of the original ones). The well known solution is given by $\hat{\mathbf{a}} = \mathbf{W}^{-1}\mathbf{F}^T(\mathbf{F}\mathbf{W}^{-1}\mathbf{F}^T)^{-1}\mathbf{f}$ and setting $(\mathbf{f}_P)_k = \psi_k(P)$ the interpolated value at any point P becomes

$$\hat{f}(P) = \sum_{k=1}^{m} \hat{a}_k \psi_k(P) \equiv \mathbf{f}_P^T \hat{\mathbf{a}} = \mathbf{f}_P^T \mathbf{W}^{-1}\mathbf{F}^T(\mathbf{F}\mathbf{W}^{-1}\mathbf{F}^T)^{-1}\mathbf{f}\,.$$

If the two point function $k(P,Q) = \sum_{k=1}^{m} W_{kk}^{-1} \psi_k(P)\psi_k(Q)$ is introduced the interpolated value takes the form $f(P) = \mathbf{k}_P^T \mathbf{K}^{-1}\mathbf{f}$ where $K_{ij} = k(P_i, P_j)$ and $(\mathbf{k}_P)_i = k(P, P_i)$. This solution can be obtained on the basis only of the function $k(P,Q)$ without specifying the base functions and the weight matrix. However if we set $W_{kk} = 1/\sigma_k^2$ and interpret σ_k^2 as the variances of the corresponding coefficients a_k, taken as zero mean random numbers, the function $f(P) = \sum_{k=1}^{m} a_k \psi_k(P) = \mathbf{f}_P^T \mathbf{a}$ becomes a zero-mean random field with covariance function

$$C(P,Q) \equiv E\{f(P)f(Q)\} = E\{\mathbf{f}_P^T \mathbf{a}\mathbf{a}^T \mathbf{f}_Q\} = \mathbf{f}_P^T E\{\mathbf{a}\mathbf{a}^T\}\mathbf{f}_Q = \mathbf{f}_P^T \mathbf{W}^{-1}\mathbf{f}_Q =$$

$$= \sum_{k=1}^{m} W_{kk}^{-1} \psi_k(P)\psi_k(Q) = k(P,Q)\,.$$

In such a stochastic interpretation the interpolated value $\hat{f}(P)$ is the prediction of the random variable $f(P)$ having minimum mean square error among all linear combinations $f_{\{\lambda_i\}}(P) = \sum_{i=1}^{n} \lambda_i f(P_i) + \gamma$ of the data, which are unbiased: $E\{f_{\{\lambda_i\}}(P)\} = E\{f(P)\}$. The optimal prediction is now given by

$$\hat{f}(P) = \mathbf{c}_P^T \mathbf{C}^{-1} \mathbf{f}\,, \qquad C_{ij} = C(P_i, P_j)\,, \qquad (\mathbf{c}_P)_i = C(P, P_i)\,. \tag{16}$$

The method for the interpolation of a single function has been already introduced in its stochastic set up by **Moritz** (1962) but it was **Krarup** (1969) who fully understood its deterministic aspects and potential and coined the name collocation for it. Introducing the "law of covariance propagation" he generalized the method to the case where observed and predicted values are not simply point values $f(P_i)$ of the function but rather the values of any continuous linear functionals $L_i(f)$. In our case where we want to predict the 4 elements of the displacement gradient matrix $\mathbf{J} = \partial \mathbf{u} / \partial \mathbf{x}$, the function f becomes in turn one of the displacement components $u_1(P)$, $u_2(P)$ and the predicted values become the linear functionals of the form $J_{mn}(P) = L_{n,P}(u_m) = [\partial u_m / \partial x_n](P)$. If the noise $\mathbf{v}$ in the data is taken into account, assumed to have zero mean and covariance matrix $\mathbf{C}_\mathbf{v}$, the optimal prediction takes the form $\hat{f}(P) = \mathbf{c}_P^T (\mathbf{C} + \mathbf{C}_\mathbf{v})^{-1} \mathbf{f}$ or in our case

$$J_{mn}(P) = \frac{\partial \mathbf{c}_P^T}{\partial x_n} (\mathbf{C} + \mathbf{C}_\mathbf{v})^{-1} \mathbf{u}\,, \qquad m, n = 1, 2\,,$$

$$C_{ik} = C_{u_m}(P_i, P_k)\,, \quad (\mathbf{c}_P)_i = C_{u_m}(P, P_i) \quad u_k = u_m(P_k)\,, \tag{17}$$

involving two covariance functions $C_{u_m}(P, Q)$ for the two displacement components $u_1(P)$, $u_2(P)$ and assuming that they are uncorrelated. Instead of the zero mean assumption for the functions $u_i(P)$ an unknown mean function can be used, which is modeled as a trend $\mu_m(P, \mathbf{x})$ involving a set of unknown parameters $\mathbf{x}$. The proper model in this case is the "least squares collocation" of **Moritz** (1978)

$$\mathbf{b} = \mathbf{A}\mathbf{x} + \mathbf{s} + \mathbf{v} \tag{18}$$

where $\mathbf{b}$ contains the observed displacements $u_m(P_i)$, $\mathbf{s}$ contains the stochastic signals $\delta u_m(P_i)$ remaining after removing the modeled by $\mathbf{Ax}$ trend $\mu_m(P_i, \mathbf{x})$ from $u_m(P_i)$ and $\mathbf{v}$ is the data noise. Under prediction is a new set of signals $\mathbf{s}'$ containing the displacement gradient elements $J_{mn} = \dfrac{\partial \delta u_m}{\partial x_n}$. It is assumed that $\mathbf{s}$, $\mathbf{s}'$ and $\mathbf{v}$ have zero means and known covariance matrices $\mathbf{C}_\mathbf{s}$, $\mathbf{C}_{\mathbf{s}'}$, $\mathbf{C}_{\mathbf{s}'\mathbf{s}}$, $\mathbf{C}_{\mathbf{sv}} = \mathbf{0}$, $\mathbf{C}_{\mathbf{s}'\mathbf{v}} = \mathbf{0}$. The solution is obtained by first solving the least squares problem $\mathbf{b} = \mathbf{A}\mathbf{x} + \mathbf{e}$, with $\mathbf{e} = \mathbf{s} + \mathbf{v}$ and weight matrix $\mathbf{P} = \mathbf{C}_\mathbf{e}^{-1} = (\mathbf{C}_\mathbf{s} + \mathbf{C}_\mathbf{v})^{-1}$ to obtain the estimate $\hat{\mathbf{x}} = (\mathbf{A}^T \mathbf{P} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{P} \mathbf{b}$ and then apply the collocation prediction to the reduced model $\mathbf{b} - \mathbf{A}\hat{\mathbf{x}} = \mathbf{s} + \mathbf{v}$ to obtain the prediction

$$\hat{\mathbf{s}}' = \mathbf{C}_{s's}(\mathbf{C}_s + \mathbf{C}_v)^{-1}(\mathbf{b} - \mathbf{A}\hat{\mathbf{x}}),\ \mathbf{C}_e = \mathbf{C}_{s'} - \mathbf{C}_{s's}[\mathbf{P} - \mathbf{PA}(\mathbf{A}^T\mathbf{PA})^{-1}\mathbf{A}^T\mathbf{P}]\mathbf{C}_{s's}^T \quad (19)$$

where $\mathbf{C}_e$ is the covariance matrix of the prediction error $\mathbf{e} = \hat{\mathbf{s}}' - \mathbf{s}'$.

Collocation was first introduced in the determination of strain parameters by **Dermanis**, **Livieratos**, **Rossikopoulos** & **Vlachos** (1981) who gave all the necessary equations for its numerical implementation and applied it to the seismic areas of Volvi in Greece and Friuli in Italy (**Bencini**, **Dermanis**, **Livieratos** & **Rossikopoulos**, 1982). Straightforward stochastic prediction in the pre-collocation sense of **Moritz** (1962) has been applied for the interpolation of vertical crustal motions (**Hein** & **Kistermann**,1981, **Fujii** & **Xia**, 1993, **El-Fiky**, **Kato** & **Fujii**, 1997). The method was again taken again into consideration rather independently in Europe (**Danuser**, **Geiger** & **Muller** 1993) and Japan (**El-Fiky**, 1998). A considerable number of applications followed in the next years: **Kahle** et al. (1995), **Straub** (1996), **Straub**, **Kahle** & **Schindler** (1997), **Kato**, **El-Fiky** & **Oware** (1997), **Kahle** et al. (1998), **El-Fiky** & **Kato** (1999), **El-Fiky**, **Kato** & **Oware** (1999), **Kahle** et al. (1999), **El-Fiky** & **Kato** (1999), **Reddy**, **El-Fiky**, **Kato**, **Shimada** & **Kumar** (2000), **Kato** & **El-Fiky** (2000), **Li**, **Miyashita**, **Kato** & **Miyazaki** (2000), **El-Fiky** (2000a), **El-Fiky** (2000b), **Kahle** et al. (2000), **Kumar**, **Miyashita** & **Li** (2002), **Caporali** (2003), **Hollenstein**, **Kahle** & **Geiger** (2006), **El-Fiky** & **Kato** (2006), **Wu**, **Tang**, **Chen** & **Li** (2006), **Tesauro**, **Hollenstein**, **Egli**, **Geiger** & **Kahle** (2006), **Egli**, **Geiger**, **Wiget** & **Kahle** (2007), **Hollenstein**, **Müller**, **Geiger** & **Kahle** (2008), **Biagi** & **Dermanis** (2009).

There are two approaches to the choice of the covariance function to be used. The Japanese researches having dense networks with many points in their disposal they compute a "sample covariance function" as a set of value pairs r_k, $C_k = (1/N_k)\sum_{r_k \le r_{PQ} < r_{k+1}} f_P f_Q$, to which an exponential model is fitted, while interpolation is carried in a planar domain. The European researches dealing with more modest networks use an inverse quadratic function $C(r) = C_0 r_0^2/(r_0^2 + r^2)$, with $C_0 = C(0)$ and a correlation length r_0 (defined by $C(r_0) = \frac{1}{2}C_0$) equal to the mean distance between the stations. **Kahle** et al. (1995) use a 3-dimensional interpolation domain and strain parameters are computed by restricting the 3-dimensional displacement gradient to the local horizontal plane. Also a quasi-spherical approach is sometimes used (e.g. **Straub**, **Kahle** & **Schindler**, 1997), **Kahle** et al., 1998) where the covariance function is expressed in terms of the spherical distance, though without taking into account the meridian convergence.

Biagi & **Dermanis** (2009) found both approaches unsatisfactory in a central Japan application, since they always give $C(r) > 0$, while the sample covariances exhibited a strong negative correlation after a certain distance. They predict geodetic coordinate velocities $\dot{\lambda} = d\lambda/dt$, $\dot{\phi} = d\phi/dt$ on the reference ellipsoid locally approximated by a best fitting sphere. Typically homogeneous and isotropic

covariances are used which are functions of the point distance. **Egli**, **Geiger**, **Wiget** & **Kahle** (2007) treat also the case where sample covariance functions appear to be inhomogeneous and anisotropic.

While collocation was developed initially for the study of the gravity field, a method for the prediction of ore reserves has been independently developed in mining geostatistics, which under the name of kriging found also application in other fields, including the study of crustal deformation (**Ayhan** et al., 2002, **van den Boogaart** & **Drobniewski**, 2002, **Zhu**, **Cai** & **Shi**, 2006, **Aktuğ** & **Kiliçoğlu**, 2006). Kriging is practically equivalent to collocation with a slight theoretical differentiation, which instead of the covariance function implements the variogram $\gamma(r) \equiv \frac{1}{2} E\{[f(P) - f(Q)]^2\} = C(0) - C(r)$ and has the (practically irrelevant) advantage that it can be applied to random fields which possess a variogram but not a covariance function. As explained by **Dermanis** (1984), both collocation and kriging produce optimal (best) predictions in the sense of the minimization of the mean square prediction error. The difference is that collocation is the "best inhomogeneous linear prediction", where optimization takes place among all predictions of the form $\Sigma_{i=1}^{n} \lambda_i f(P_i) + \gamma$, while kriging is the "best homogenously linear prediction", where optimization takes place among all predictions of the form $\Sigma_{i=1}^{n} \lambda_i f(P_i)$. For a zero mean random field f the constant term γ in collocation vanishes and both methods give identical results. Universal kriging is a variant which produces exactly the same results as the least squares collocation with a modeled trend (non-zero mean). The only potential advantage of kriging appears in the case of a constant non-zero mean $E\{f\} = \mu$, where in contrast to the case of collocation the value of μ can be unknown. A generalization of kriging, similar to that of **Krarup** (1969) to the stochastic prediction of **Moritz** (1962), has been given by **Reguzzoni**, **Sansó** & **Venuti** (2005).

7. Crustal deformation analysis beyond the plane domain

Strictly speaking crustal deformation of the earth is a three-dimensional process just like the deformation of any material body. When we use planar methods in crustal deformation we do not refer to the deformation of any actual material plane surface but rather to the deformation of an abstract plane formed by projecting points at the surface of the earth on a horizontal reference surface.

Realizing this shortcoming some authors tried to apply three-dimensional deformation methods to the analysis of geodetic deformations. **Brunner** (1979), has proposed the analysis of the 3-dimensional displacement gradient $\mathbf{J} = \frac{1}{2}(\mathbf{J} + \mathbf{J}^T) + \frac{1}{2}(\mathbf{J} - \mathbf{J}^T) = \mathbf{E} + \mathbf{\Omega}$ into a symmetric (infinitesimal strain) part $\mathbf{E}$ and an antisymmetric part $\mathbf{\Omega} = [\boldsymbol{\omega}\times]$ having $\boldsymbol{\omega}$ as its axial vector. The infinitesimal strain part contains the "extensions in the directions of the axes" $\varepsilon_i = E_{ii}$ in its di-

agonal and the "shearing strain components" $\gamma_{ik} = 2E_{ik}$ outside the diagonal. The displacements $\mathbf{u}_i$ produced from the coordinate variations in a 3-dimensional network are assumed to be linear functions of the (approximate) coordinates $\mathbf{x}_i$, i.e., $\mathbf{u}_i = \mathbf{J}\mathbf{x}_i = \mathbf{C}_i\mathbf{s}$ over the network area, easily expressed as linear functions of the 9 strain parameters $\mathbf{s} = [\varepsilon_1\ \varepsilon_2\ \varepsilon_3\ \ \tfrac{1}{2}\gamma_{12}\ \tfrac{1}{2}\gamma_{23}\ \tfrac{1}{2}\gamma_{13}\ \ \omega_1\ \omega_2\ \omega_3]^T$. For all points P_i, taking also errors $\mathbf{v}_\mathbf{u}$ into account, the strain parameters are estimated from the least squares solution to the total adjustment model $\mathbf{u} = \mathbf{C}\mathbf{s} + \mathbf{v}_\mathbf{u}$. The use of 3-dimensional networks and deformation parameters was also suggested by **Reilly** (1982, 1987, 1990), **Pietrantonio** & **Riguzzi** (2004). A 3-dimensional stain analysis in the framework of a 3-dimensional finite element method with quadrilateral elements has been used by **Kiamehr** & **Sjöberg** (2005).

The problem with 3-dimensional crustal deformation analysis is that discrete observation data are limited to the 2-dimensional surface of the earth. While interpolation in the horizontal sense is meaningful and can be in principle improved by network densification, the extraction of 3-dimansional strain parameters requires an extrapolation in the third vertical direction which is not supported by the data at hand. However it is the most relevant approach in engineering applications when man-made structures are monitored by networks that extend equally in both the horizontal and vertical direction.

The planar approach despite the abstract (no material) character of the deforming surface is justifiable from a geophysical point of view. Geophysical processes affecting horizontal motions are of a different nature compared to those affecting vertical ones and in this respect it makes sense to study horizontal deformation separately from height variations as in fact is routinely done.

The planar approximation is satisfactory when the network is of small size so that the variation of coordinates on a cartographic plane can be used. This is permissible since the deformation from the actual horizontal surface, the ellipsoid, to the cartographic plane under a "locally centered" map projection is negligible for small areas. For more extensive networks the curvature of ellipsoid or its spherical approximation must be rather used. The problem has been rigorously solved by **Biagi** & **Dermanis** (2009) who using appropriate differential geometry tools determined that the deformation gradient $\mathbf{F} = \mathbf{I} + \mathbf{J}$ to be used for the usual 2-dimensional strain analysis at any point on a reference ellipsoid (with major semi-axis a and eccentricity e) is given by

$$\mathbf{F} = \begin{bmatrix} \dfrac{M_0}{M}\dfrac{\partial\lambda}{\partial\lambda_0} & \dfrac{M_0}{N\cos\phi}\dfrac{\partial\lambda}{\partial\phi_0} \\ \dfrac{N_0\cos\phi_0}{M}\dfrac{\partial\phi}{\partial\lambda_0} & \dfrac{N_0\cos\phi_0}{N\cos\phi}\dfrac{\partial\phi}{\partial\phi_0} \end{bmatrix} \tag{20}$$

where λ_0, ϕ_0 are the geodetic coordinates at the initial epoch t_0 and λ, ϕ the coordinates at a later epoch t, $M = M(\phi) = a(1-e^2)(1-e^2\sin^2\phi)^{-3/2}$, $N = N(\phi) = a(-e^2\sin^2\phi)^{-1/2}$, $M_0 = M(\phi_0)$ and $N_0 = N(\phi_0)$. **Pope** (1966) (see also **Snay** & **Cline**, 1980) gave formulas for the computational of the strain parameters which (apart from an obvious sign error) are equivalent to the use of the matrix

$$\mathbf{F} = \begin{bmatrix} \dfrac{\partial\lambda}{\partial\lambda_0} & \dfrac{1}{\cos\bar{\phi}}\dfrac{\partial\lambda}{\partial\phi_0} \\ \cos\bar{\phi}\dfrac{\partial\phi}{\partial\lambda_0} & \dfrac{\partial\phi}{\partial\phi_0} \end{bmatrix} \tag{21}$$

$\bar{\phi}$ being the mid-latitude of the network region. **Drew** & **Snay** (1989) gave formulas, which are equivalent to the use of the matrix

$$\mathbf{F} = \begin{bmatrix} \dfrac{\partial\lambda}{\partial\lambda_0} & \dfrac{\bar{N}\cos\bar{\phi}}{\bar{M}}\dfrac{\partial\lambda}{\partial\phi_0} \\ \dfrac{\bar{M}}{\bar{N}\cos\bar{\phi}}\dfrac{\partial\phi}{\partial\lambda_0} & \dfrac{\partial\phi}{\partial\phi_0} \end{bmatrix} \tag{22}$$

with $\bar{N} = N(\bar{\phi})$, $\bar{M} = M(\bar{\phi})$, $\bar{\phi}$ being the latitude of the "reference" origin. They do not give any clue to how their equations were derived or any related reference to the literature.

Savage, **Gan** & **Svarc** (2001) start from the 3-dimensional case relating displacements u_λ, u_ϕ, u_r in a local spherical system (east, north, zenith) to the spherical components of the strain tensor $\varepsilon_{\lambda\lambda}$, $\varepsilon_{\lambda\varphi}$, $\varepsilon_{\lambda r}$, $\varepsilon_{\varphi\varphi}$, $\varepsilon_{\varphi r}$, ε_{rr} and the rotation vector ω_λ, ω_φ, ω_r and arrive by restriction to the spherical surface to an expression (eq. A6) of the form

$$\begin{bmatrix} u_\lambda \\ u_\varphi \end{bmatrix} = \begin{bmatrix} \bar{r}\cos\bar{\varphi}(\lambda-\lambda_0) \\ \bar{r}(\varphi-\varphi_0) \end{bmatrix} = \begin{bmatrix} a_\lambda(\omega_\lambda,\omega_\varphi) \\ a_\varphi(\omega_\lambda,\omega_\varphi) \end{bmatrix} + \begin{bmatrix} \varepsilon_{\lambda\lambda} & \varepsilon_{\lambda\varphi}-\omega_r \\ \varepsilon_{\lambda\varphi}+\omega_r & \varepsilon_{\varphi\varphi} \end{bmatrix} \begin{bmatrix} \bar{r}\cos\bar{\varphi}(\lambda-\bar{\lambda}) \\ \bar{r}(\varphi-\bar{\varphi}) \end{bmatrix}$$

$$= \begin{bmatrix} a_\lambda \\ a_\varphi \end{bmatrix} + \mathbf{J} \begin{bmatrix} \bar{r}\cos\bar{\varphi}(\lambda-\bar{\lambda}) \\ \bar{r}(\varphi-\bar{\varphi}) \end{bmatrix} \tag{23}$$

where $\bar{\lambda}$, $\bar{\varphi}$, $\bar{r}$ are coordinates of the network midpoint. The strain parameters are derived from the displacement gradient $\mathbf{J} = \mathbf{F} - \mathbf{I}$, which corresponds to the choice of $\mathbf{F}$ given by eq. (21). The same approach has been used by **Savage**, **Svarc** & **Prescott** (2004), **Hammond** & **Thatcher** (2004, 2007). Note that in addition to the strain components and the rotation ω_r within the spherical horizontal

plane, the rotation components ω_λ, ω_φ are also simultaneously estimated, which give rise to the "within the sphere" translations a_λ, a_φ, i.e. to a rigid rotation around an Euler vector.

Another alternative, based on the fact that observations are limited on the earth surface, is the computation of strain parameters of the natural earth surface conceived as a 2-dimensional material shell. This approach has been introduced by **Altiner** (1996, 1999) and **Voosoghi** (2000) and has been applied to several studies, e.g. **Haas**, **Gueguen**, **Scherneck**, **Nothnagel** & **Campbell** (2000), **Hefty** & **Duraciova** (2002), **Grafarend** & **Voosoghi** (2003), **Klek**, **Ragowski** & **Jarosinski** (2003), **Altiner**, **Marjanovic**, **Medved** & **Rasic** (2006), **Moghtases-Azar** (2007), **Moghtased-Azar** & **Grafarend** (2009). The method has the advantage that it incorporates simultaneously height variation information but has a serious defect from the geophysical interpretation point of view. Indeed crustal deformation remains a 3-dimensional process and the strain parameters derived by the method correspond to a section of the crust by a plane tangent to the surface at each particular point rather than to an independently deforming 2-dimensional material shell. Thus a trace of the actual 3-dimensional deformation is looked at from a direction (surface normal) which is different at any surface point, especially in regions with highly undulating terrain. On the contrary the classical horizontal treatment looks at a trace of the 3-dimensional deformation from an always vertical direction (normal to the plane, sphere or ellipsoid), i.e. from a direction compatible with the separation of the geophysical causes into those affecting horizontal motion (e.g. plate and tectonic block motions) and those affecting vertical motion (e.g. postglacial uplift or hydrological effects).

8. Separating deformation from relative motion

When analyzing observations it is essential to separate the relative motion of separate tectonic blocks from the deformation within each block. Such block-related deformation can be expressed either by strain parameters considered constant over the block, or by strain parameter fields varying continuously over the block when a displacement interpolation model is used. This reflects the typical inefficiency of standard interpolation methods such as collocation to represent discontinuities in the data and they must hence be applied in a piecewise manner. The need to separate relative motion from deformation had already been realized by **Whitten** (1960) (see also **Whitten & Claire**, 1960) who analyzed fault crossing observations using a model combining a shear in the fault direction with a displacement of one fault side with respect to the other. **Chrzanowski** was the one to strongly emphasize the separation of rigid motion from deformation (see e.g. **Chrzanowski**, **Chen**, & **Secord**, 1983) in contrast to typical cross-faulting applica-

tions where the derived large strains reflected a relative slippage along the fault rather than a pure deformation.

While rigid motion in the planar case is represented by 2 translations and 1 rotation, in the study of larger areas analysis must be carried on the surface, where the equivalent rigid motion (at least for geologically small time intervals) is a rotation around a fixed pole (unit vector $\vec{n} = \vec{n}(\lambda_P, \phi_P)$) with constant angular rate ω. The corresponding rigid motion parameters are either the 3 Cartesian components of rotation vector $\vec{\omega} = \omega\vec{n}$, usually referred as "Euler vector", or its spherical components λ_P, ϕ_P, ω. Euler vectors over different sub-regions can be determined by a least squares fitting so that the residual velocity components after the removal of the rigid rotation of the whole block are minimized (see e.g. **Koyama**, 1996, **Zhou** et al., 1997, **McClusky** et al., 2000, **McClusky** et al., 2003, **Bock** et al., 2003, **Shen**, **Lü**, **Wang** & **Bürgmann**, 2005, **Nishimura** & **Hashimoto**, 2006). In addition to the a-priori removal of such rigid rotations it is possible to estimate them simultaneously with the strain related signals in a unified data analysis procedure. For example in collocation the components of Euler rotation vectors over different tectonic blocks can be incorporated in the trend parameters $\mathbf{x}$ of the least squares collocation model $\mathbf{b} = \mathbf{A}\mathbf{x} + \mathbf{s} + \mathbf{v}$ (18). It is also possible to consider three-dimensional rigid or even similarity transformations (3 rotations, 3 translations, 1 scale factor) in order to remove rigid motion trends from the 3-dimensional velocities produced in GPS networks, either a-priori (**Rossikopoulos**, **Fotiou**, **Livieratos** & **Baldi**, 1998) or simultaneously within a data analysis procedure such as the least squares collocation (**Kahle** et al., 1995). A simultaneous estimation of the strain rate components in the tangent plane and the Euler vector has beed presented by **Ward** (1998a, 1998b).

There is absolutely no reason for a rigid motion within a sphere to have the form represented by a constant Euler vector with fixed pole and constant angular velocity. In the most general case the rigid motion may take place around a migrating pole with variable angular velocity. Realizing this fact **Biagi** & **Dermanis** (2009) introduced the concept of a different "discrete Tisserant" reference system for each separate tectonic block. It is defined by keeping constant the barycenter of the sub-network of the block and minimizing the relative kinetic energy of its points (visualized as mass points with the same mass) with respect to the new system or equivalently by requiring that the relative angular momentum vector is vanishing. If there is a scale defect the mean quadratic length L of the sub-network (defined by $L^2 = \sum_i \mathbf{x}_i^T \mathbf{x}_i$) must be kept constant. Thus the original block motion is separated into a rigid motion of the whole Tisserand reference system around a time dependent Euler vector $\vec{\omega}(t)$ and a deformation with respect to the Tisserand system which represents in an optimal way the block sub-network as a whole. A version of the same concept for applications on the plane has been presented by **Biagi** & **Dermanis** (2006) and **Dermanis** & **Kotsakis** (2006).

9. Quality assessment of strain parameters

There are two problems when formal covariance propagation is used to produce variances and covariances of strain parameters from those of the input data. The first is relating to the presence of additional modelling errors arising by the differences between the actual displacements or velocities and their interpolated predictions. Note that strain parameters have the character of spatial derivatives and thus continuous knowledge of the displacement field must be in hand by some interpolation method. The influence of this "interpolation" error is varying from complete negligence in the case of the finite element method, to improper reflectance in the a-posteriori variance when the linear displacement model is used over sub-regions and finally to its proper incorporation in the collocation method provided that the true covariance is known. Since the final assumption is certainly not fulfilled, the reliability of the produced error covariances for the elements of the displacement (or velocity) gradient matrix depends on the validity of a series of assumptions: the zero mean assumption (i.e. the proper removal or modelling of the signal trend), the assumption of homogeneity and isotropy of the displacement or velocity random field and finally the ability to have a good sample covariance function from the data itself, which is possible only for very dense network with a large number of stations.

A second problem is the fact that strain parameters are nonlinear functions of the elements of the strain matrix, especially the eigenvalues. The same holds true for the singular values and the rigorous strain parameters of **Biagi** & **Dermanis** (2006, 2009). As a consequence covariance propagation based on linearization may not produce the correct covariances and even produce additional biases. (**Livieratos** & **Vlachos**, 1981, **Soler** & **van Gelder**, 1991, **Xu** & **Grafarend**, 1996, **Han**, **van Gelder** & **Soler**, 2007, **Cai** & **Grafarend**, 2007a, 2007b).

Covariance propagation after linearization has the advantage that is free of any hypothesis about the distribution of the errors in the input data. Within a "second order" estimation and prediction methodology only the covariance and the (zero) means of the input data are required. Alternative methods for covariance propagation are Monte Carlo methods , which do not require a linearization. They have benn used for crustal starins by Jouanne, Menard & Jault (1994), Martinod et al. (1996) and Jouanne, Hippolyte, Gamond & Martinod (2001). Nevertheless, just like the tests of statistical hypotheses, they require complete knowledge of the probability distribution of the input data. Such knowledge is hardly ever at hand and researchers tend to resort to one of the most widespread scientific myths: that of the Gaussian distribution as a proper model for practically every type of data errors.

A final question concerns the credibility of the covariances of the input data as well as the hypothesis that the relevant estimates of displacements or velocities (which are the input data in many analysis approaches) are unbiased, i.e. free from

the influence of neglected systematic errors. Since nowadays crustal deformation studies are based of velocity estimates from permanent GPS stations their input covariances should not be the formal software-produced covariances of daily or weekly solutions, which are typically over-optimistic. Analysis of the produced coordinate time series demonstrates variations well above the formal noise level as well as strong periodic signals especially in the vertical component. Furthermore the noise in the series is not white but coloured with correlations in nearby epochs. There is a very large literature on the topic of obtaining realistic covariance matrices of GPS station velocities but it lies outside the scope of the present work.

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