Modeling the Seismogenic Process of Earthquake Occurrence in the Black Sea Region Using Statistical Methods

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Abstract: In this study we will use both Cornell and extreme values (Gumbel I) distributions to model the seismogenic process for all the earthquake sources located around and across the Black Sea region and to compute all the quantities used in probabilistic hazard assessment. The differences in methods have effects in results distribution and the challenge is finding the most appropriate method to be used. The geological information about the Black Sea Basin were taken from Tari, et al. (2000), Meredith et al. (2002). The observational seismic data base were obtained using both macroseismic (1901-1933) and instrumental data (1934-2010) and the earthquake catalogues for each source were downloaded from the EMSC site. The final seismic hazard results obtained during this study should not be interpreted as the only possible versions. Since the used models are based on probabilistic models, rather than on factual knowledge, it is impossible to present results that shows the general truth.

1. Introduction

When one is speaking about natural hazards an increasing amount of attention is paid to economic questions and questions of public safety. Lomnitz (1976) distinguishes between questions of public safety (loss of life) and questions of economic loss, considering that loss of life represents the primary risk and that the economic losses are secondary. Risk of life in earthquakes and especially in tsunamigenic earthquakes is closely correlated with the largest possible magnitude and the recurrence time of this disasters.

There are several methods for the estimation of earthquake risk through which Lomnitz (1976) distinguishes two basic levels of approach in all type of analysis:

- "according to the depth of statistical treatment: descriptive statistics, or statistical inference";
- "according to the degree of interaction with physical description of the underlying process: stochastic models, or models which incorporate geophysical results of a deterministic nature".

Each of this level represents a stage of sophistication in the analysis, rather than a true difference in approach. Lacking both adequate data and an adequate model of the physical mechanism, the statistician will try to extract a maximum of information from all available sources. In the words of a UNESCO Report (1972), "the amount of relevant data concerning natural hazards is inadequate for the immediate application of standard methods and statistical inference".

The application of statistical methods to the estimation or prediction of such hazards offers the hope of reducing the loss of life and the damage caused by natural disasters. Different statistical methods and models (Bayesian, optimization, composition, zoning or simulation methods) and the way to use them in seismology and earthquakes study are extensively presented by different authors, namely Lomnitz (1976) and Moldovan (2007).

2. The methods

In our study we will present and use only the extreme-values methods (Gumbel I and Gumbel III) – Gumbel (1958) and models and the probabilistic method used by Cornell (1968) for seismic hazard assessment. The differences in methods have effects in results distribution and the challenge is finding the most appropriate method to be used.

The practical advantage of using Cornell method is the multitude of numerical programs existing nowadays: EQRisk - (McGuire, 1976), SEISRISK III (Bender and Perkins, 1987), FRISK88M (Risk Engineering, 1996), CRISIS99 (Ordaz, 1999). The ability to incorporate to incorporate geologic information through the definition of seismic source zones appears to be another advantage offered by the Cornell-McGuire method as compared to other statistical methods. At present the Cornell-McGuire method is the most widely used method for site specific hazard analysis worlwide.

Some of the more important practical advantages of extreme-value methods are given by Lomnitz (1976):

– The extreme values of a geophysical variable are better known, more homogeneous in time and more accurately determined than the average events in a time series of data;
– The method does not require a detailed knowledge of the parent distribution;
– The method is simple to use and to understand. It involves few assumptions and the uncertainties are relatively easy to discuss.

This advantages have their own risks and dangers, e.g. the extrapolation beyond the range of values for which data is available. Sources of errors exists in all statistical studies of geophysical phenomena, because the data series analysis are based on the assumption of stationarity, that is affected for long term studies by the secular variations of variables. Long term correlations in the data, including secular effects, introduce non-conservative errors in statistical methods based on data stationarity.

In this study we will use both Cornell and extreme values (Gumbel I) distributions to model the seismogenic process for all the earthquake sources located around and across the Black Sea region and to compute all the quantities used in probabilistic hazard assessment.

The final seismic hazard results obtained during this study should not be interpreted as the only possible versions. Since the used models are based on probabilistic models, rather than on factual knowledge, it is impossible to present results that shows the general truth.

3. Observational Data and Results

For this study we have used the Black Sea seismotectonic model given by Minshull et al. (2005), taking into account both seismic and geophysical data. The zonation model used for seismic hazard analysis by both statistical methods, as well as the location of of the seismogenic sources around Black Sea are presented in Figure 3.1. The geographic distribution of the studied sources is also presented in Table 1.

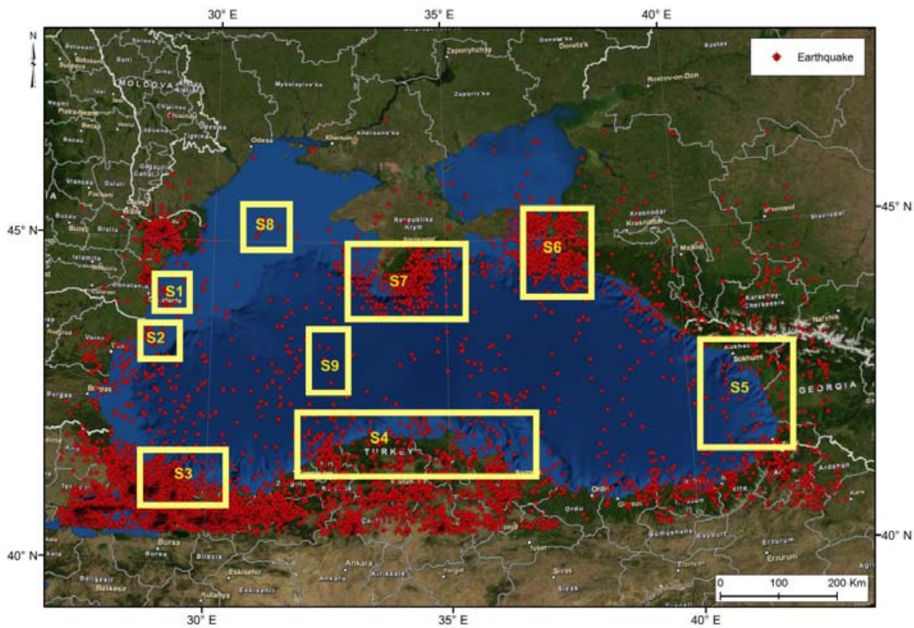

Figure 3.1. The earthquakes distribution along Black Sea areal and position of each seismic sources.

The geological information about the Black Sea Basin were taken from Tari, et al. (2000), Meredith et al. (2002). The observational seismic data base were obtained using both macroseismic (1901-1933) and instrumental data (1934-2010) and the earthquake catalogues for each source were downloaded from the EMSC site.

Table 1. The geographic distribution of Black Sea seismic sources

Seismic Sources	Coordinates		Seismic Sources	Coordinates	
Central Dobrogea S1	43.883	27.407	Novorossjsk S6	44.30	36.75
	44.784	27.407		45.25	36.75
	43.883	28.663		45.24	38.15
	44.784	28.663		44.30	38.18
Shabla S2	43.00	28.50	Crimeea S7	43.75	32.75
	43.74	28.50		45.00	32.75
	43.74	29.50		45.00	35.00
	43.00	28.50		43.75	35.00
Istanbul S3	41.00	28.60	West Black Sea S8	45.00	30.00
	41.60	28.60		45.50	30.00
	41.60	30.00		45.50	31.35
	41.00	28.60		45.00	31.35
North Anatolian Fault S4	41.50	32.00	Mid Black Sea S9	42.50	31.00
	42.25	32.00		43.50	12.00
	42.25	35.75		43.50	32.00
	41.50	35.75		42.50	32.00
Georgia S5	41.00	40.00	Background	40.00	28.00
	43.25	40.00		46.00	42.00
	43.25	42.00		40.00	42.00
	41.00	42.00		46.00	28.00

At a simple look at the catalogues one can see that the seismicity might be characterized as irregular and week, with very rarely strong events followed by years without any activity, or by years with a slightly increased seismicity. The week seismic activity characterizing the marine earthquakes from the Black Sea basin, the reduced number of large events, the inconsistent catalogues makes the statistical studies to be very difficult to interpret and to give a reliable conclusion.

The irregularity of the Black Sea recorded seismicity might be associated with the week coverage with seismic stations of the marine basin. Nowadays we don't stat if the irregularity cause is due to weak activity stroke once in a wile by large earthquakes with tsunamigenic potential or by the lack of seismic stations around the Black Sea.

From the 9 seismic sources we have extensively studied 7 with the Cornell method and 5 with the extreme values method. Even if at the beginning of our studies we intended to use both extreme value distributions (Gumbel I – GI and Gumbel III – GIII) during the computation process we have concluded that the existing catalogues can not be used for GIII studies because the particular characteristics of the seismicity patterns.

For the last studies we have chosen only the seismic sources with tsunamigenic potential (S1, S2, S4, S6 and S7). S3 source is an inland source while S8 and S9 produce so small and rare earthquakes, that they might be included in the background seismicity.

Cornell - McGuire Statistical Method Used For The Seismogenic Process Modeling In The Black Sea Areal

In Table 2 are presented the characteristics of each source from the Black Sea that will be used in the seismic hazard assessment with the Cornell method. Because S8 and S9 are very low risk seismic sources and will not be further analyzed.

Table 2. Input parameters for probabilistic hazard assessment using crustal sources from the Black Sea basin

Sources	Computing Coordinates		Average depth (km)	Mmin used	Mmax observed	Mmax Computed	b	β_M	Seismic activity rate
S1	44.00	27.50	11	3.0	4.6	5.6	0.65	0.43078	0.11864
S2	43.00	28.50	16.4	3.0	7.2	6.7	0.32	1.13943	0.16513
S3	41.00	29.00	22.1	3.0	6.7	7.2	0.53	0.63488	0.47761
S4	41.50	32.00	14.8	3.0	6.1	6.3	0.61	0.4943	0.74074
S5	41.00	40.00	13.5	3.0	5.5	5.7	0.59	0.52763	1.03921
S6	44.30	36.75	20.8	3.0	5.2	5.8	0.75	0.28768	0.59091
S7	43.75	32.75	22.8	3.0	6.5	6.3	0.38	0.96758	0.25301
S8	45.00	30.00	14.8	3.0	4.9	-	X	X	0.19512
S9	42.50	31.00	26.9	3.0	3.9	-	0.72	0.3285	0.25581
B.grd			15.8	3.0	5.0	-	0.61	0.4943	2.42446

With the input data set from Table 2, we have applied the algorithm of Cornell - McGuire, 1976 and the EqRisk program to compute the seismic hazard parameters for S1-S7 seismic sources, such as: the number of events with a given magnitude per year, the return periods for different magnitudes, the annual hazard, the hazard for 50, 100, 475 and 1000 years. Using numerical computations we have also obtained the maximum possible magnitude for each zone (column 6 from Table 2).
We observe that the computed values are in good correlation with the maximum observed magnitudes being included in the ±0.5 degrees of magnitude. Only for S1 the difference is +1.0. The return periods seems to be very large and far from those expected. As an example in Table 3 are the return periods for Mw = 6 for sources S1-S7.

Table 3. The return periods for Mw=6 for sources S1-S7

Source	S1	S2	S3	S4	S5	S6	S7
Tr (years)	>10000	1422	134	2778	>10000	>10000	3717

In figures 3.2-3.4 we have represented de dependence of the expected magnitude versus the return period and the hazard curves for sources S1 to S7.

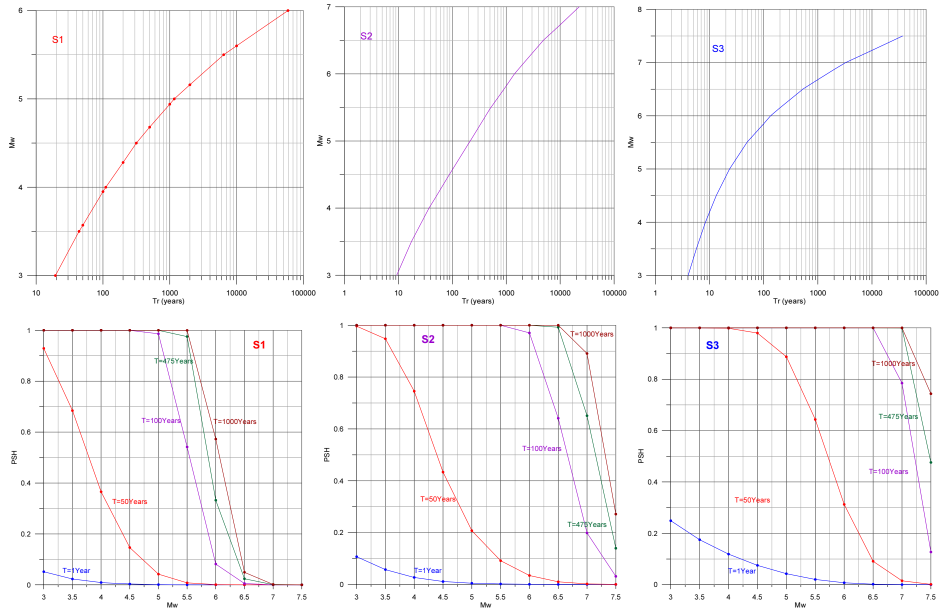

Figure 3.2. Return periods for earthquakes with different magnitudes (up) and the hazard curves for different exposure periods (down) for seismic sources S1 – S3

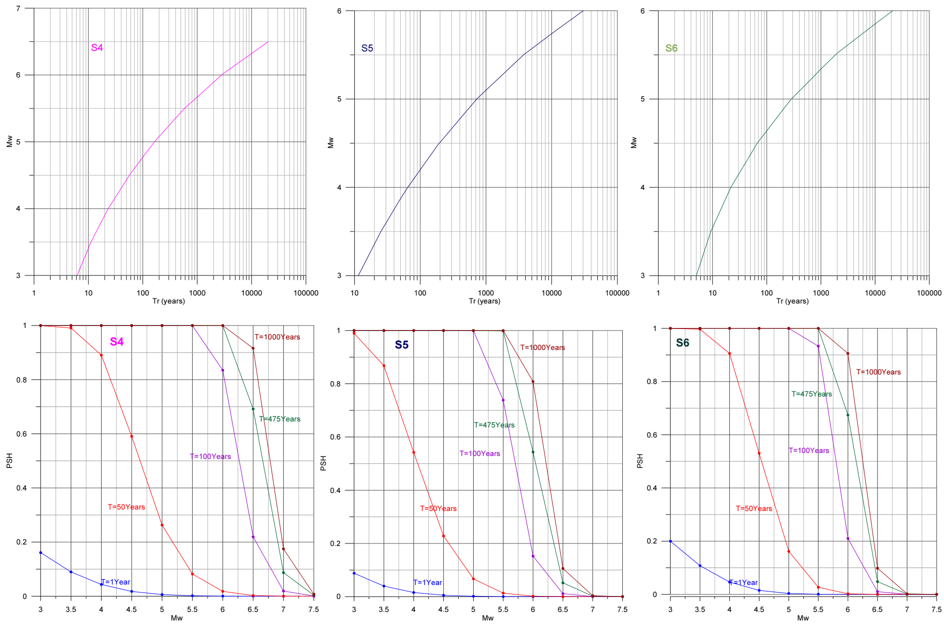

Figure 3.3. Return periods for earthquakes with different magnitudes (up) and the hazard curves for different exposure periods (down) for seismic sources S4-S6

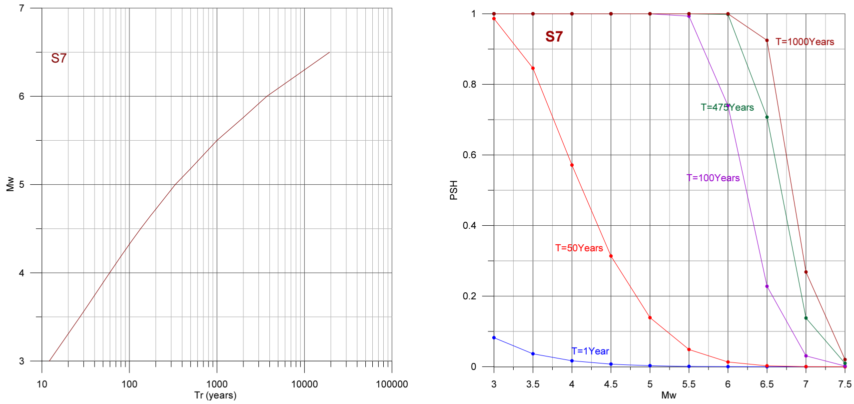

Figure 3.4. Return periods for earthquakes with different magnitudes (left) and the hazard curves for different exposure periods (right) for seismic source S7

Extreme Values Gumbel I (GI) Statistical Method Used For The Seismogenic Process Modeling In The Black Sea Areal

Let α be the mean number of earthquakes per year above magnitude zero. Than y, the maximum annual earthquake magnitude, will be distributed as in eq. 3.1., Gumbel's (1958) formula of the first distribution (GI):

$$G(y) = e^{-ae^{-by}} \quad \text{with} \;\; y \geq 0 \tag{3.1}$$

In order to estimate the parameters α and β one takes the largest yearly earthquake magnitudes y_1, y_2, y_3, y_4 ... y_n in a sample of n consecutive years. These magnitudes are than arranged in order of increasing size, so that $y(1) \leq y(2) \leq \ldots \leq y(n)$. $G(y)$ becomes:

$$G(y(j)) = \frac{j}{n+1} \tag{3.2}$$

Finally, the values of α and β are estimated from the least-square fit to eq (3.1):

$$\log[-\log G(y)] = \log \alpha - \beta y \tag{3.3}$$

The extreme magnitudes for seismic sources S1, S2, S4, S6 and S7, containing 34/108/54/44/83 years of recordings but only 28/17/36/22/25 years with useful data were used to obtain the GI statistical distribution:

$G(M\max)$ and $\ln[-\ln G(M\max(j))]$

as a function of Mw (Figures 3.5) and than with the means of least squares program the coefficients: $\ln\alpha$, $\ln\beta$, a, b and u (Table 4).

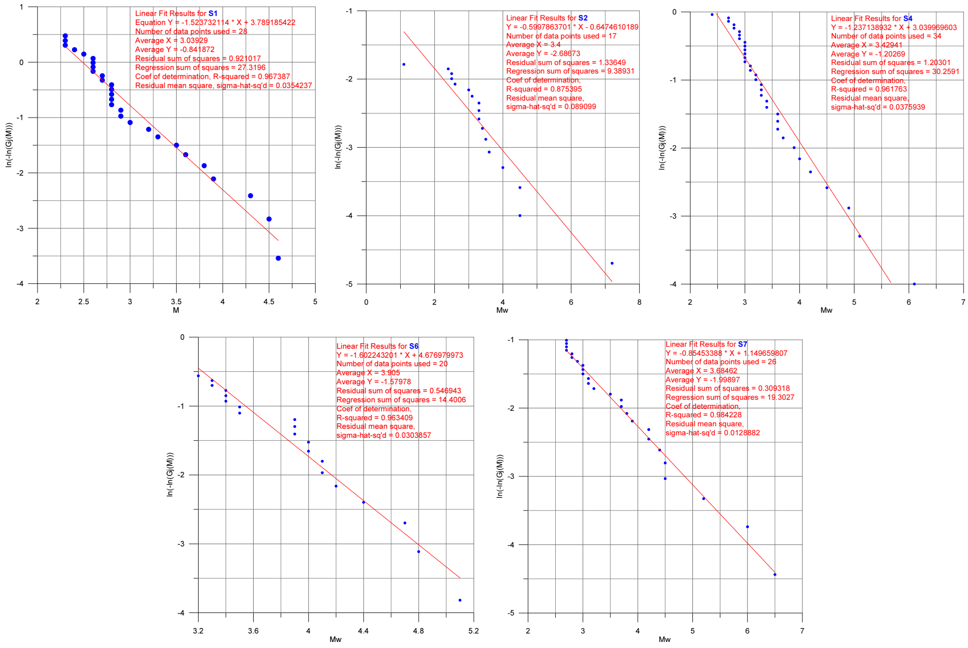

Figure 3.5. Gumbel's type I distribution for S1,S2,S4,S6 and S7 earthquakes

Table 4. The main GI parameters for the studied sources

S	β	$\ln\alpha$	α	u	$a = \ln(\alpha)/\ln 10$	$b = \beta/\ln 10$
S1	1.523732	3.789185	44.22036	2.486779	1.645622	0.661748
S2	0.599786	-0.647461	0.523373	1.079486	0.281189	0.260484
S4	1.237139	3.03997	20.90461	2.457258	1.320242	0.537283
S6	1.602243	4.67698	107.4451	2.91902	2.031187	0.695845
S7	0.854534	1.14966	3.157119	1.345365	0.499291	0.371119

Once the parameters α and β have been determined (Table 4), there can be obtained the following quantities used in probabilistic hazard assessment (Epstein and Lomnitz, 1966):

1. Mean magnitude of earthquakes in a region

If Mmin is the magnitude threshold of an earthquake catalog, the mean magnitude is estimated by:

$$\bar{M} = M\min + \frac{1}{\beta} \tag{3.4}$$

2. Number of shocks above magnitude Mmin:

The yearly number of earthquakes above magnitude zero is α. Hence the expected number of shocks above magnitude $M_{\min}$ in D years is:

$$DNy = Dae^{-\beta M \min} \tag{3.5}$$

3. Mean return period

If N is the expected number of earthquakes per year, $T = 1/N$ is the mean return period in N years.

For example, the mean return period for shocks exceeding magnitude y is:

$$Ty = \frac{1}{Ny} = e^{(\beta y)}/\alpha \tag{3.6}$$

4. Modal maxima

The modal annual maximum $\tilde{y}$ is that maximum which is most frequently observed. It is not the mean of all maxima, but rather the maximum which has the highest probability of occurrence, i.e. which makes dG/dy a maximum:

$$\tilde{y} = \frac{\log x}{\beta} \tag{3.7}$$

The mean return period of magnitude $\tilde{y}$ is exactly one year.

5. Exceedance probability

The probability that a given magnitude y be exceeded during any given year is:

$$P\{Y \geq y\} = 1 - G(y) \tag{3.8}$$

6. Occurrences with specified probability

The value of the earthquake magnitude which is exceeded with probability p in a D year period is given by:

$$y_p(D) = y_p + \frac{1}{\beta}\log D \tag{3.9}$$

where:

$$y_p = \tilde{y} - \beta^{-1}\log[-\log(1-p)] \tag{3.10}$$

is the annual maximum exceeded with probability p.

The earthquake hazard $H_D(y)$ is the probability of occurrence of an earthquake of magnitude y or more in a D year period:

$$H_D(y) = 1 - e^{-aDe^{-\beta y}} \tag{3.11}$$

Once the parameters from Table 4 have been determined for all the sources we have also computed the quantities given in equations (3.4) to (3.7). The results of computations are presented in Table 5.

Table 5. The Mean magnitude of earthquakes in a region, the Number of shocks above magnitude Mmin in 1 year, the Mean return period and the Modal maxima for sources S1, S2, S4, S6 and S7

Source	Mmin	$M = M\min + \frac{1}{\beta}$ (Eq. 3.4)	$Ny = \alpha e^{-\beta M\min}$ (Eq. 3.5)	$Ty = \frac{1}{Ny} = e^{(\beta y)}/\alpha$ (Eq. 3.6)	$\overline{y} = \frac{\log\alpha}{\beta}$ (Eq. 3.7)
S1	1.523732	2.3	2.95628339	1.32923251	0.75231383
S2	0.599786	1.1	2.76726132	0.27056972	3.69590510
S4	1.237139	1.1	1.90831661	5.36081925	0.18653865
S6	1.602243	2.6	3.22412505	1.66720441	0.59980647
S7	0.854534	2.7	3.87022845	0.31424514	3.18222901

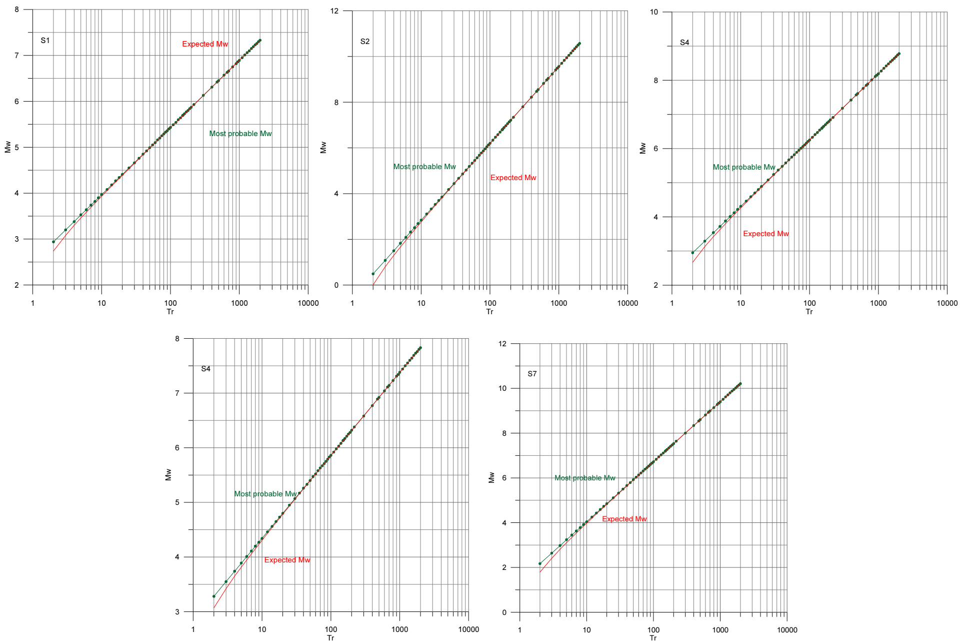

Figure 3.7. Most probable and the expected magnitude as function of the return period Tr for S1, S2, S4, S6 and S7.

Using all the computed values and a numerical iteration algorithm we have obtained: the return periods, the most probable and the expected magnitudes as function of their return periods (Figure 3.6 and 3.7), the probability of exceedance and finally the Probabilistic Seismic Hazard (PSH) for 4 exposure periods of 1, 50, 100 and 475 years (Figures 3.8)

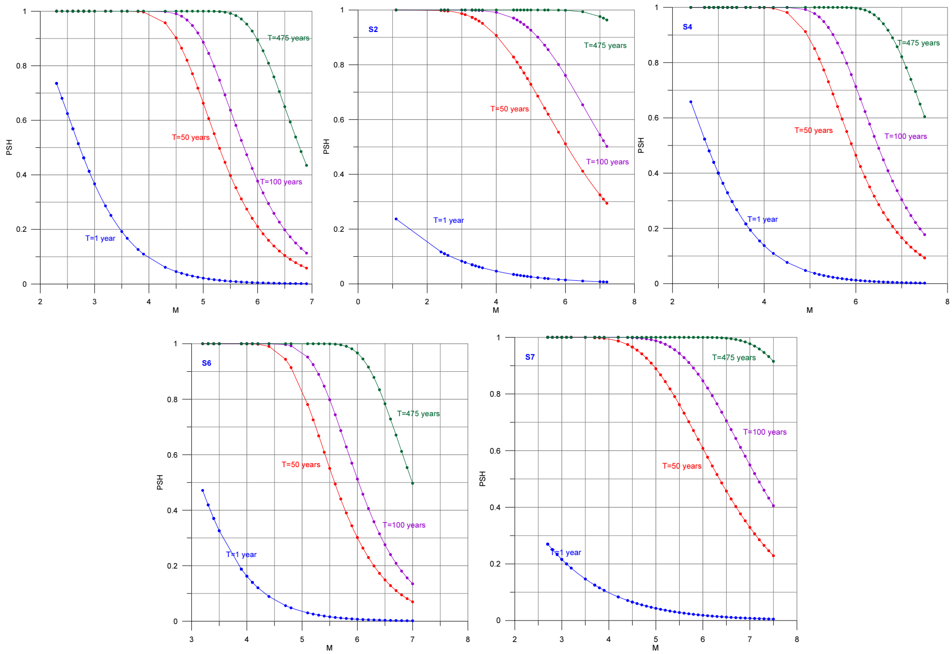

Figure 3.8. Hazard curves for S1, S2, S4, S6 and S7

4. Conclusions

During this study we have obtained the PSH curves and the return periods of different magnitudes for the seismic sources from the Black sea Basin, using two analyzing methods (Cornell and Gumbel). With the first analyzing method (Cornell) the return periods of different magnitudes seems to be large in comparison with the return periods obtained using the second statistical processing method (Gumbel). As an example in Table 6 are the return periods for Mw=6 for sources S1-S7 for both analyzing methods.

Table 6. The return periods for Mw=6 for sources S1-S7

Source	S1	S2	S3	S4	S5	S6	S7
Tr (years) Cornell	>10000	1422	134	2778	>10000	>10000	3717
Tr (years) GI	250	90	-	95	-	130	55

Another credible explanation of this differences is given by the earthquake catalogs for all this sources, catalogues that reveal the low earthquake potential of the

Table 7. The return periods for Mw=6 for sources S1-S7

Source	S1	S2	S3	S4	S5	S6	S7
Mmax (1000 years) Cornell	5.6	6.7	7.2	6.3	5.7	5.8	6.3
Mmax (1000 years) GI	6.4	8.4	-	7.5	-	6.8	8.5
Mmax observed	4.6	7.2	6.7	6.1	5.5	5.2	6.5

sources and also the bad coverage with recordings systems of the Black Sea basin leading to inconsistent catalogues. The solution for this issue is a joint seismic monitoring of the Black Sea basin, involving all countries around the sea. The maximum expected magnitude obtained with both methods for the studied seismic sources are presented in Table 7 together with the maximum observed magnitude.

As we expected, the magnitude values for the GI method are very high in comparison with those obtained usig the Cornell method and also with those observed. The explanation is the same as for the low values of return periods, i.e. the working hypothesis without upper limit for the magnitude.That's why the Gumbel III extreme values distribution studies should be needed for the sources from Black Sea Basin. Unfortunately the existing catalogues does not permit this type of numerical statistical analysis.

The final conclusion is that for a reliable statistical seismic analysis of the Black Sea areal is needed a common and uniform seismic monitoring for all states along the sea coast.

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